STAR Note #64

Expressions for Diffusion in Radial Drift TPC's

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Abstract

We derive an expression for the size of the charge cloud diffusing (and drifting) in a radial $(1/r)E$ field. Calculations using this expression, Monte Carlo, and an approximate expression are compared for a prototype radial TPC for STAR, as well as for a larger chamber.

A prototype radial TPC is under development (by a BNL-UCD-LBL collaboration) for measurements of higher η particles than will be measured by the central TPC[1] of the STAR (Solenoidal Tracker at RHIC) detector.[2] These external radial TPC's would employ radial drift (essentially perpendicular to the beam directions) and have the advantage (over an array of rectangular[3] TPC's) of azimuthal symmetry as well as improved multitrack spatial resolution in the high-hit-density region near the beam pipe.

The better azimuthal spatial resolution of a radial TPC is due to the magnification of space in the azimuthal direction as the track charge clouds drift to larger radii. One must also consider the effects of diffusion. A Monte Carlo program (described in ref.[4]) has been written to simulate drift and diffusion of a 3-D charge cloud structure so that its size in $(r,\ z,\ \phi)$ at any point and its time profile at some collection radius r can be given. The r or field dependence of both the drift velocity and the diffusion coefficient, usually called σ_1,[5] can be included as can the effects due to stray magnetic or fringe electric fields. The Monte Carlo results presented in Fig. 1 (top) show that after a 35 cm drift the ratio of the track separation to the track charge cloud width is larger for a radial TPC that for a conventional (rectangular) TPC.

Here we present the derivation of a closed expression for the azimuthal size of the charge cloud drifting and diffusing in a radial $(1/r)$ field. This expression is analytic (can be solved) if $\sigma_1(r)/r$ is integrable, and reduces to a simple expression when $\sigma_1(r)$ is represented by a polynomial in r. The size in the other directions (r and z, in cylindrical coordinates) can, of course, also easily be calculated. Here we also compare analytic predictions to the Monte Carlo calculations for two gases under consideration for the radial prototype and a larger chamber.

We assume that the "hit" at $r = r_0$ has an initial electron charge distribution which is well-described by a gaussian of standard deviation σ_0 in the transverse direction. The azimuthal size of the cloud at any radius, r, includes both the effects of diffusion and of radial magnification. In a radial distance dr the latter is given by $1 + dr/r$. In this distance, we assume that due to diffusion each tiny charge cloudlet grows by an amount $\sigma_1\sqrt{dr}$, and that the diffusion and radial drift are independent processes. Then the variances (σ^2) at r and $r + dr$ are related by

$$\sigma^2(r + dr) = \sigma^2(r)(1 + dr/r)^2 + \sigma_1^2(r)dr$$

or to first order in dr

$$\sigma^2(r + dr) - \sigma^2(r) = 2\sigma^2(r)dr/r + \sigma_1^2(r)dr.$$

With $y = \sigma^2$, the above becomes $dy/dr = 2y/r + \sigma_1(r)^2$. Trying $y = r^2 f(r)$, the final solution is

$$\sigma(r) \equiv \sigma_\phi(r) = r\left[(\sigma_0/r_0)^2 + \int_{r_0}^{r} (\sigma_1(r)/r)^2 dr\right]^{\frac{1}{2}} \tag{1}$$

where σ_ϕ is the SD in the azimuthal or ϕ direction. In the radial direction the SD of the cloud is

$$\sigma_r(r) = \left[\sigma_0^2 + \int_{r_0}^{r} \sigma_1^2(r)dr\right]^{\frac{1}{2}}.$$

The physics of the solution (1) is best displayed by assuming the diffusion coefficient $\sigma_1(r)$ is independent of r. Then for $\sigma_1(r) = \sigma_1$

$$\sigma(r) = r\left[(\sigma_0/r_0)^2 + \sigma_1^2(r - r_0/rr_0\right]^{\frac{1}{2}}. \tag{2}$$

One can, in most cases, (except for short drift distances) neglect the first term. (In our prototype, we will be dealing mainly with minimum ionizing particle tracks, and r_0 varies from 12.5 to 47.6 cm.) Then[6]

$$\sigma(r) \simeq \sigma_1\sqrt{(r - r_0)r/r_0}. \tag{3}$$

Two tracks (hits) at $r = r_0$, but separated in azimuth by D initially, are separated in centroid by rD/r_0 at r after drifting a distance $x = r - r_0$.

For σ_1 =constant, the ratio of final separation to width is $(r/r_0)^{\frac{1}{2}}(D/\sigma_1\sqrt{x})$ and improves as $(r/r_0)^{\frac{1}{2}}$ over the case of conventional (rectangular) drift. As noted earlier, Figure 1 (top) shows a Monte Carlo of the case of radial drift as compared to rectangular drift. The cloud profiles are shown for initial values of $\sigma_0 \simeq 10\mu m$. Figure 1 (bottom) defines and plots a "figure of merit" for the two cases and shows the advantages of the radial TPC for hits near the beam pipe (where the density is largest) and which (in the figure) have the largest drift distances.

In Fig. 2 the points show values of σ_ϕ(47.6 cm), the azimuthal SD of the charge cloud at the final radius of 47.6 cm as calculated by Monte Carlo (MC). The values from eqn. 1 with σ_1 approximated by a linear r dependence (see Fig. 3) are given by the line and agree well with the Monte Carlo calculations. The dashed lines show the predictions of the approximate solutions with σ_1 constant and chosen to give a best "fit" to the Monte Carlo points. This approximation is good if one can guess the "fit" σ_1 and if (as is the case here) σ_1 is not changing much as a function of r ($i.e$ field). The abscissa are the values of the starting (hit) radii, r_0. The plots labelled BNL gas are for the mixture 80% Ar, 15% Isobutane and 5% Methylal. That labelled P10 is 90% Ar and 10% CH_4. Figure 4 consists of similar plots for a larger chamber with inner (cathode) and collector (anode) radii of 50 and 200 cm. The BNL gas has $\sigma_1 \simeq 250\mu m$, about a factor of two smaller than those for P10 and thus smaller $\sigma_\phi(r)$ values. The P10 gas has the advantage of a much lower operating voltage.

In summary, an exact expression for the diffusion of a charge cloud in a radial ($E \sim 1/r$) electric field has been given and the advantages of radial drift have been demonstrated. With a judicious choice of σ_1 =constant this exact expression reduces to a simple one which gives a good approximation to "exact" predictions and displays the essential features of the physics of radial drift.

We thank Daniel Cebra for introducing us to the problem, James Draper for his helpful comments, and Gergely Zimanyi for his assistance in solving the differential equation.

References

[1] Daniel Cebra *et al.*, Proposal for RHIC Detector R & D, FY92.

[2] Update to RHIC LOI for An Experiment on Particle and Jet Production at Midrapidity, LBL-31040.

[3] Ken Foley *et al.*, STAR Collaboration Meeting, March 2–6, 1992 at LBL.

[4] Juan Romero, Paul Brady, and Daniel Cebra, *et al.*, STAR Note #65.

[5] Drift and Diffusion of Electrons in Gases: A Compilation, Anna Peisert and Fabio Sauli, CERN 84–08 (1984).

[6] This result was first obtained by James E. Draper. In STAR Note #45, he was able to evaluate the width $w(r)$ for the case of σ_1 =constant and $\sigma_0 = 0$ using the limit of a discrete sum, and he obtained this expression (3) if his $w(r)$ is interpreted as the SD, $\sigma(r)$ of the distribution. Expression (2) follows when initial conditions are included.

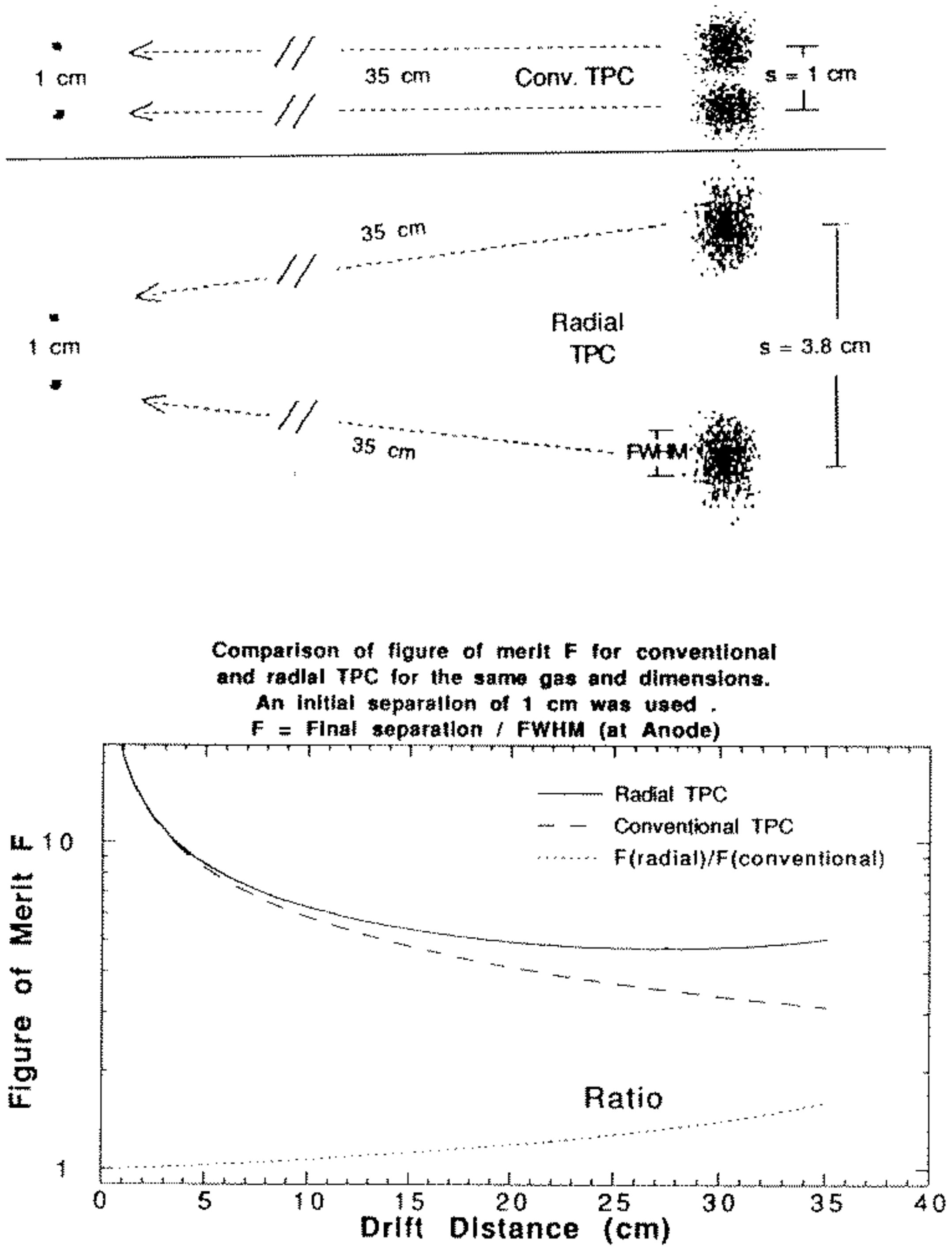


Figure 1: (top): A Monte Carlo of rectangular (conventional) and radial drift and diffusion over 35 cm; (bottom): figure of merit, F =centroid separation/FWHM both at collecting anode vs starting or drift distance for conventional and radial TPC.

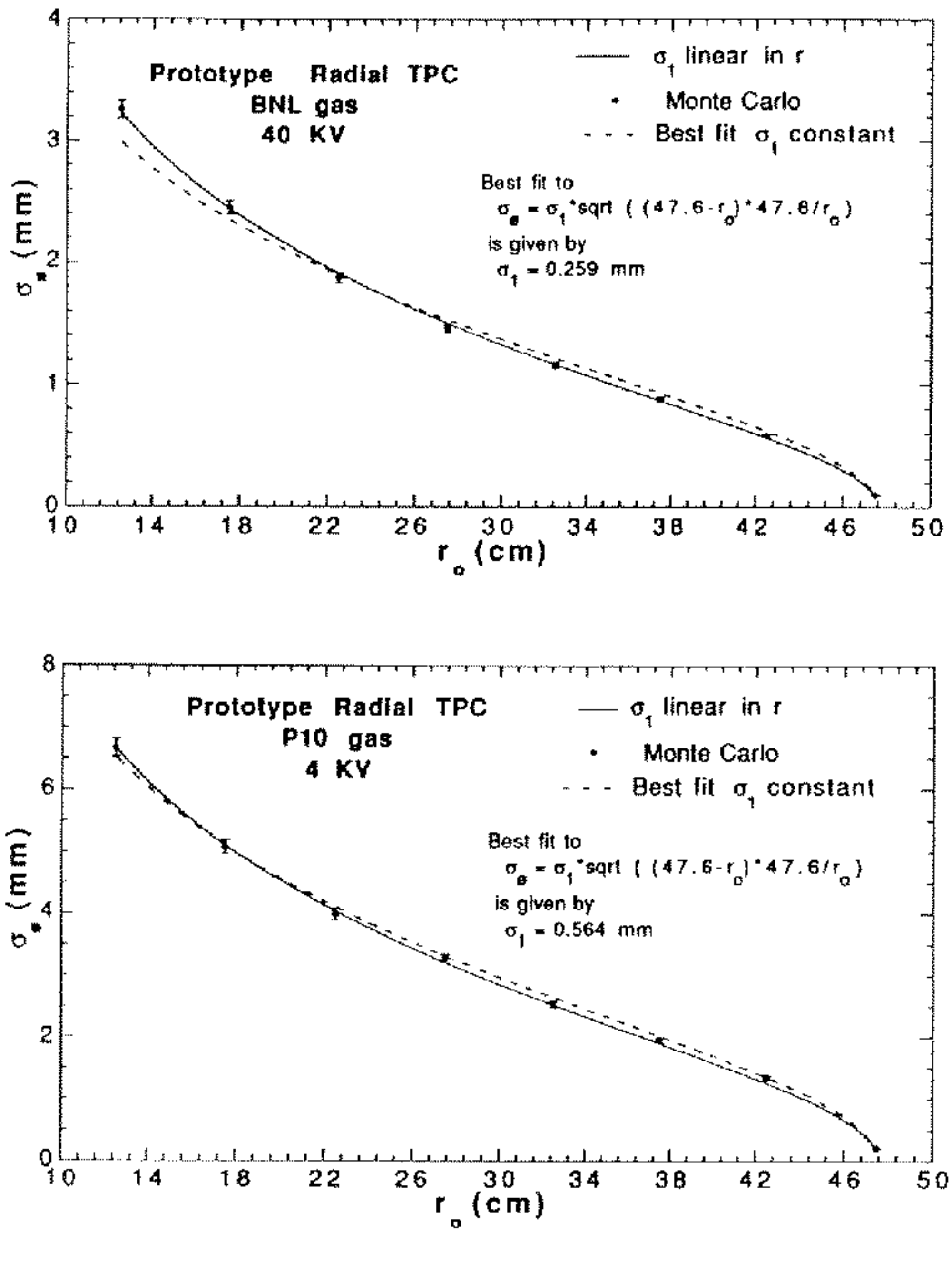
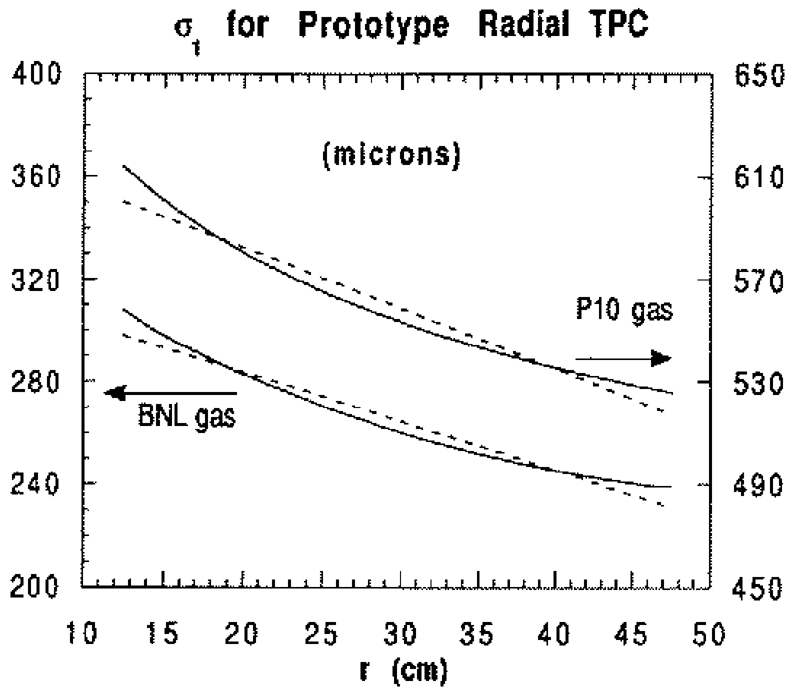


Figure 2: Comparisions of the azimuthal size of the charge cloud vs starting radius, r_0, in the prototype radial TPC. The points are calculated by Monte Carlo, the solid line via the analytical expression (Eq. 1) with $\sigma(r) = a + br$ and the dashed line is with σ_1 =constant, but adjusted to fit the points (Eq. 3). Top: BNL gas (80% Ar, 15% Isobutane, and 5% methylal) operated at 40 KV ($a = 0.0322$ cm, $b = -0.00192$ cm^{-1}). Bottom: P10 gas (90% Ar, 10% Methane), operated at 4KV ($a = 0.0628$ cm, $b = -0.00237$ cm^{-1}).



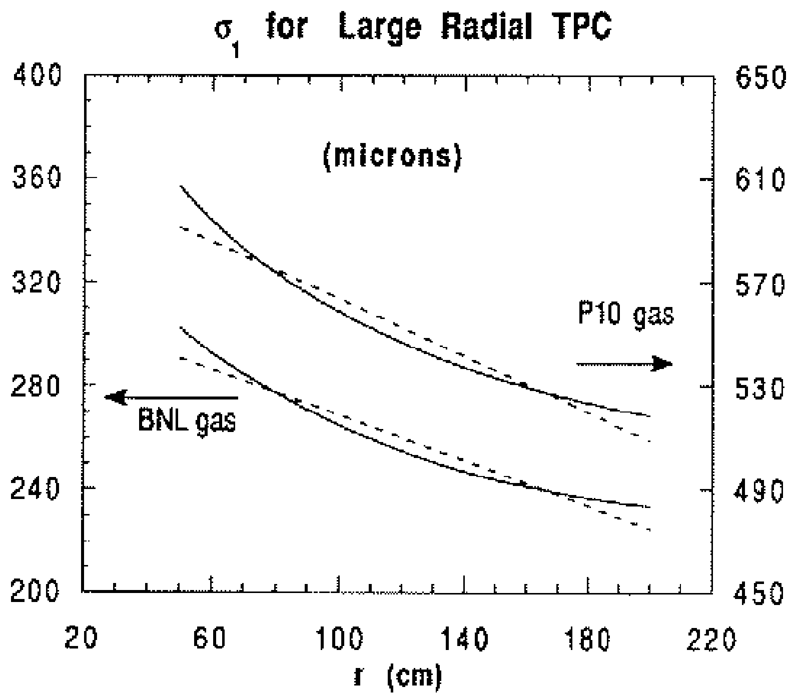


Figure 3: Radial dependence of σ_1 for prototype radial TPC (top) and large radial TPC (bottom). Left (right) vertical axis corresponds to BNL (P10) gas. Dotted lines are the best fits using $\sigma_1 = a + br$.

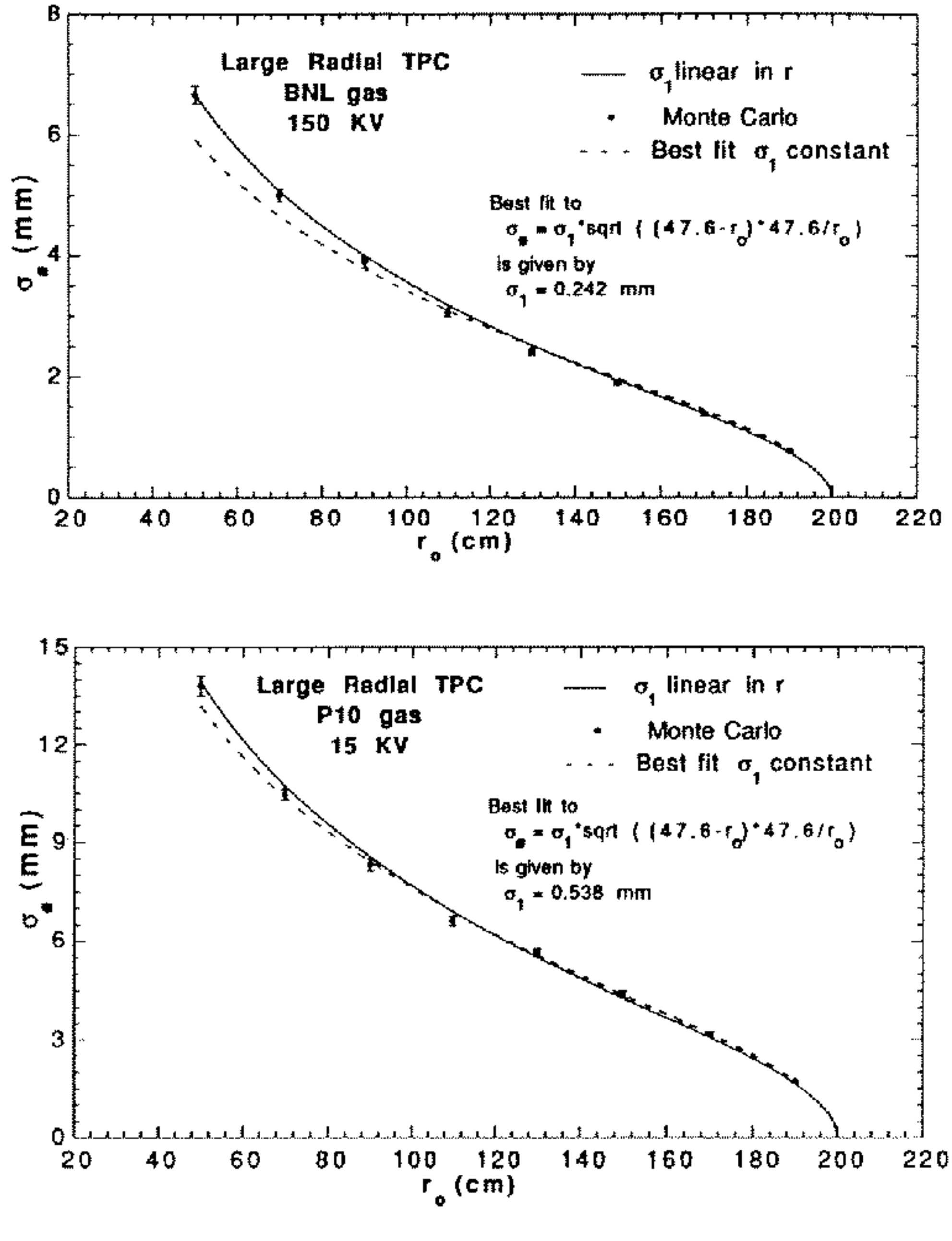


Figure 4: As described in Fig. 2 for a larger radial TPC. Top: BNL gas operated at 150 KV (a = 0.0313 cm, $b = -0.000439$ cm^{-1}). Bottom: P10 gas opertaed at 15 KV (a = 0.0618 cm. $b = -0.000548$ cm^{-1}).