SPIN-PARITY ANALYSIS OF ρ' IN THE REACTION $\gamma p \rightarrow \pi^+ \pi^+ \pi^- \pi^- p$ AT 9 TO 18 GeV

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Abstract: We develop a formalism concerning the four-pion decay of a resonance including identical particles. A simple test based on this formalism when applied to new high-energy data on the reaction $\gamma p \to \rho' p \to \pi^+ \pi^+ \pi^- \pi^- p$ yields strong evidence that ρ' is a vector meson (1⁻), although in the data examined we cannot exclude another possibility, $J^P = 2^+$.

1. Introduction

In this paper we develop a formalism relevant to the four-pion decay of a resonance including identical particles. By means of a simple test based on this formalism and with new higher energy data we obtain strong evidence that the recently discovered four-pion resonance*, called ρ' as in ref. [4], with a mass of 1.6 \pm 0.1 GeV and a width of 0.5 \pm 0.1 GeV [2] in the reaction

$$\gamma p \to \pi^+ \pi^+ \pi^- \pi^- p \tag{1}$$

is a vector meson (1⁻). All other choices for the spin-parity, namely 0^{\pm} , 1^{+} , 2^{-} and higher spins are ruled out with the exception of 2^{+} which is possible but appears unwarranted in the data examined.

The data used are from the general photoproduction experiment carried out at the Stanford Linear Accelerator Center using the 2 m streamer chamber with a hydrogen gas target exposed to an 18 GeV bremsstrahlung beam. The experimental procedures [6], analysis of the reaction $\gamma p \rightarrow \pi^+\pi^- p$ [7] and other aspects of reaction (1), ref. [8], have been described elsewhere. The present analysis deals with 1256.9 weighted (979 raw) events of the reaction (1) at incident energies in the range 9-18 GeV corresponding to the total cross section of $4.4\pm0.2~\mu b$, which is approximately constant throughout this energy range.

^{*} First indication for this resonance was obtained by Davier et al., refs. [1, 2] in the preliminary data of the experiment described here. Since then two other experiments have obtained supporting evidence, namely Barbarino et al. [3] in the reaction $e^+e^- \rightarrow 4\pi$ and Bingham et al. [4] in reaction (1) at 9.3 GeV with a linearly polarized beam. A theoretical implication of this meson has been discussed by Bramon and Greco [5].

2. Formalism

We consider a resonance R with spin-parity j^η decaying into $\rho^0\pi^+\pi^-$ with subsequent decay of ρ^0 into $\pi^+\pi^-$ as in fig. 1. In the rest frame of R let R_i stand for the rotation giving the orientation of the three-particle final state $\rho\pi\pi$ with respect to a standard one and similarly in the rest frame of ρ let S_i be a rotation giving the two-particle final state from the decay of ρ . To specify R_i we need three (Euler) angles, whereas for S_i only two angles. Let w_i be a set of three (Dalitz) variables, e.g. pion energies in the R rest frame, which are further necessary to fix the four-particle final state up to rotations R_i and S_i . The subscript i refers to one for the four diagrams of fig. 1 arising from interchanges of identical particles.

Now the amplitude for the decay of R from an angular momentum eigenstate (j, m) can be written as*

$$M_{m} = \sum_{ik\lambda} B_{k\lambda}(w_{i}) D_{mk}^{j*} (R_{i}) D_{\lambda 0}^{1*} (S_{i}) , \qquad (2)$$

where λ is the helicity of ρ , k is the projection of the angular momentum j onto an analyzer axis w (defined later), m is the same onto a standard axis z, and the index i runs through the four diagrams of fig. 1 so that the total amplitude is symmetric with respect to interchange of identical particles.

The eq. (2) is a usual result and if the width of ρ were narrow or the data were predominantly in the ρ non-overlap regions of the Dalitz plot, then for these events we could identify a unique pair of pions in the ρ band and consequently avoid the problem of symmetrization. In practice the ρ has a wide width and the mass of R is so low that it is often even kinematically impossible to identify a unique pair of pions ascribable to ρ . Thus we are forced to the fully symmetric amplitude of eq. (2), which unfortunately does not even tell what angular distributions to look at. We solve this problem in the following way.

Let R be any rotation of the three-body final state $\rho\pi\pi$. Because we are dealing with a group, rotations R^{-1} and $R^{-1}R_i$ (denoted r_i henceforth) exist so that using the representation property of the D-functions we can rewrite eq. (2) in the form

$$M_{m} = \sum_{l} A_{l} D_{ml}^{j*}(R) , \qquad (3)$$

where we have defined ampltitudes

$$A_{l} = \sum_{ik\lambda} B_{k\lambda}(w_{i}) D_{\lambda 0}^{1*}(S_{i}) D_{lk}^{j*}(r_{i}).$$
 (4)

^{*} To make the description more transparent we write $D^j_{mk}(R)$ meaning the customary Wigner D-function notation $D^j_{mk}(\alpha\beta\gamma)$ where the Euler angles $\alpha\beta\gamma$ parameterize the rotation R. Eq. (2) follows from the basic formalism as described, for example, in ref. [9]. A hint for how to proceed further was obtained from ref. [10] dealing with A_1 decay.

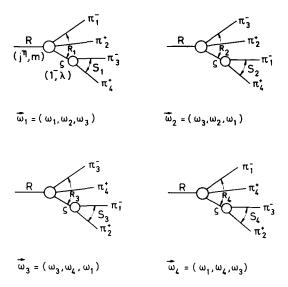


Fig. 1. Four diagrams of ρ' decay into $\pi^+\pi^+\pi^-\pi^-$ arising from permutations of identical particles.

The reduced amplitudes A_l depend on five (three Dalitz and two angular) variables which are necessary to fix the four-body final state up to rotation R.

Among many choices (continuum!) that are possible for the rotations R and R_i we make the following choices because they turn out to be more suitable for our application. In the rest frame of R let e_i for the unit vector along π_i . We define the axis w to be a unit vector bisecting e_1 and e_3 , and v to be the normal to the plane containing e_1 and e_3 , and obviously v to be $v \times w$. v is the rotation which, when applied to the standard axes v, v, v, gives the analyzer (or body) axes v, v, v just defined. That is in terms of Euler angles v, v, v is such that v are the azimuth and polar angles of v with respect to v, v, v and v is the angle of final rotation about v. Similarly the rotation v is specified by choosing the analyzer axes v is v and v is the obtained by appropriate interchanges v and v and v are then obtained by appropriate interchanges v and v and v as indicated in fig. 1.

Once R and R_i are given, the rotations r_i follow from the geometry. Prameterizing r_i in terms of Euler angles α_i , β_i and γ_i we find that the above choices for R and R_i imply

$$\alpha_1 = \alpha_3 = 0, \quad \alpha_2 = \alpha_4 = \pi, \tag{5a}$$

$$\beta_1 = \beta_2 = \beta_3 = \beta_4 = \theta$$
, (5b)

where 2θ is the opening angle between e_1 and e_3 , and

$$\gamma_1 = \tan^{-1} \frac{(e_1 \times e_2) \cdot e_3}{(e_1 \times e_2) \cdot (e_3 \times e_1)}$$
 (5c)

with the rest of $\gamma_{i'}$ s obtained by making corresponding permutations of particle indices in eq. (5c).

The ρ decay is treated as usual. We take in the rest frame of ρ one of the pion momenta to be the equivalent w axis. The relevant rotation S_i is then parameterized by teh azimuth and polar angles ψ_i and χ_i , with the corresponding angle γ taken to be zero.

The definition of the standard state or the choice of the standard axes, x, y, z are likewise free to some extent. For the decay of R we choose them to be the s-channel helicity axes, that is, z is along the direction of R in the overall c.m. (or opposite to the outgoing proton in the R rest frame), y is the normal to the production plane, $\gamma \times z$, and, of course, x is $y \times z$. With this choice the meaning of m in eqs. (2) and (3) is the helicity of R. Similarly for the decay of ρ , z is taken to be along the direction of ρ in the rest frame and y normal to the ρ production plane so that γ in these equations stand for the helicity of ρ .

The emission of ρ is analogous to radiative transitions in nuclear physics. And thus it is convenient to introduce amplitudes

$$L_{k}(w_{i}, S_{i}) = B_{k,0}(w_{i}) d_{0,0}^{1}(\chi_{i}),$$
(6a)

$$E_k(w_i, S_i) = [B_{k,1}(w_i) - B_{k-1}(w_i)]\cos \psi_i d_{10}^1(\chi_i),$$
 (6b)

$$M_k(w_i, S_i) = [B_{k,1}(w_i) + B_{k-1}(w_i)] \sin \psi_i d_{10}^1(\chi_i),$$
 (6c)

in terms of which eq. (4) can be written as

$$A_{l} = \sum_{ik} (L_{k} + E_{k} + iM_{k}) D_{lk}^{j*}(r_{i}).$$
 (7)

As apparent in their definitions the electric (E_k) and magnetic (M_k) amplitudes give rise to a transversely polarized ρ , whereas the longitudinal ones (L_k) give rise to a longitudinally polarized ρ . One can now show that parity conversation implies*

$$(L, E)_{-k} = \epsilon (-1)^k (L, E)_k$$
, (8a)

$$M_{-k} = \epsilon (-1)^{k+1} M_k , \qquad (8b)$$

where ϵ stands for $\eta(-1)^{j}$.

Finally the last D-function, a geometrical factor, appearing in eq. (7) may also be treated in a similar manner. Substituting eq. (5) and recombining terms we define amplitudes with a definite parity property in the following way:

$$E_{l}^{+} = \sum_{i} \left[E_{0} d_{l0}^{j}(\theta) + \sum_{k>0}^{j} e^{il\alpha_{i}} \left[E_{k} d_{lk}^{j}(\theta) + E_{-k} d_{l-k}(\theta) \right] \cos k\gamma_{i} \right], \tag{9a}$$

^{*} With our choice of the standard axes parity conservation implies $B_{-k-\lambda} = \epsilon(-1)^{-k-\lambda} B_{k\lambda}$ as discussed in ref. [8]. Eq. (8) follows simply from this result.

$$E_{l}^{-} = \sum_{i} \sum_{k>0}^{j} e^{il\alpha_{i}} [E_{k} d_{lk}^{j}(\theta) - E_{-k} d_{l-k}^{j}(\theta)] \sin k\gamma_{i}, \qquad (9b)$$

and likewise for the other types of amplitudes. Eq. (7) can now be written as

$$A_{I} = (L_{I}^{+} + E_{I}^{+} - M_{I}^{-}) + i(L_{I}^{-} + E_{I}^{-} + M_{I}^{+}).$$
(10)

For a given type + and - amplitudes behave like opposite parity objects. Using eq. (8) it is seen that parity conservation implies

$$(L^+, E^+, M^-)_{-l} = \epsilon(-1)^l (L^+, E^+, M^-)_l,$$
 (11a)

$$(L^-, E^-, M^+)_{-l} = \epsilon(-1)^{l+1} (L^-, E^-, M^+)_l$$
 (11b)

The angular distribution can now be written in a standard way as

$$\frac{\partial W}{\partial \alpha \partial \cos \beta} = \frac{2j+1}{4\pi} \sum_{mnl} |A_l|^2 \rho_{mn} D_{ml}^{j*} (\alpha \beta 0) D_{nl}^j (\alpha \beta 0) , \qquad (12)$$

where we have introduced a density matrix ρ for the production of R. In particular assuming s-channel helicity conservation, that is, taking $\rho_{11} = \rho_{-1-1} = \frac{1}{2}$ and all other diagonal elements zero, the cos β projection takes the explicit forms

$$W(x) = \frac{3}{4} \left[a_0 (1 - x^2) + a_1 (1 + x^2) \right] , \tag{13a}$$

$$W(x) = \frac{5}{4} \left[a_0 3x^2 (1 - x^2) + a_1 (4x^4 - 3x^2 + 1) + a_2 (1 - x^2) (1 + x^2) \right], \quad (13b)$$

$$W(x) = \frac{7}{32} \left[a_0 3(1 - x^2)(5x^2 - 1)^2 + a_1 \frac{1}{2} (225x^6 - 305x^4 + 111x^2 + 1) \right]$$

$$+a_25(1-x^2)(9x^4-2x^2+1)+a_3\frac{15}{2}(1-x^2)^2(1+x^2)$$
, (13c)

for j = 1, 2 and 3 respectively, where we have written x for $\cos \beta$ and a_l for $|A_l|^2$. In the above expression the absolute square of amplitudes can be taken as

$$a_{l} = |A_{l}|^{2} = |L_{l}^{+} + E_{l}^{+} - M_{l}^{-}|^{2} + |L_{l}^{-} + E_{l}^{-} + M_{l}^{+}|^{2}$$
(14)

without the cross term because the latter is odd in *l*. In addition because of the normalization these decay parameters may be taken to satisfy

$$a_0 + 2\sum_{l>0}^{j} a_l = 1. ag{15}$$

Thus far we have only pursued the consequences of rotation invariance and parity conservation. In order to apply this formalism to the data in its full extent further assumptions are necessary. In particular one has to specify how the amplitudes like $B_{k\lambda}(w_i)$ depend on the Dalitz variables w_i . This dependence will certainly involve various momenta raised to some powers (related to the orbital angular momentum) having to do with the relevant decay vertices as well as the (Breit-Wigner-like) propa-

gator for ρ . Only after such model assumptions are made can one proceed to calculate the sum over the four diagrams of fig. 1 as in eq. (9).

We note however the following simple feature in our formalism giving rise to a model-independent test for the spin-parity. As implied by eq. (5c) and momentum conservation every $\sin \gamma_i = 0$ for events for which all four pions are in one plane. Consequently because of eq. (9b) all — type amplitudes vanish for these events. In addition due to the $\sin \psi_i$ factor in eq. (6c) magnetic amplitudes vanish so that only L^+ and E^+ amplitudes can lead to coplanar decays. Thus as implied by eq. (11a) the presence of l=0 term in the $\cos \beta$ distribution for coplanar events will rule out the spin-parity series with $\epsilon=-1$, i.e. 0^- , 1^+ , 2^- , This indeed happens to be a useful test in our data as we show in the following.

3. Data

The reaction (1) in the energy range 9-18 GeV is dominated by the production of ρ^0 and Δ^{++} to the extend of about 60% and 25% of total, respectively. As discussed in ref. [2] the mass histogram (fig. 2) of the four-pion system in this reaction exhibits a broad enhancement which is centered at 1.6 ± 0.1 GeV and has a width of 0.5 ± 0.1 GeV. That this ρ' decays essentially all to $\rho^0 \pi^+ \pi^-$ events is evident from the shaded spectrum of fig. 2, which corresponds to events involving at least one ρ (defined by the mass band $0.68 < m(\pi^+ \pi^-) < 0.84$ GeV). To emphasize this ρ' effect we also show in fig. 2 the peripheral $\rho \pi \pi$ phase space taking into account the forward

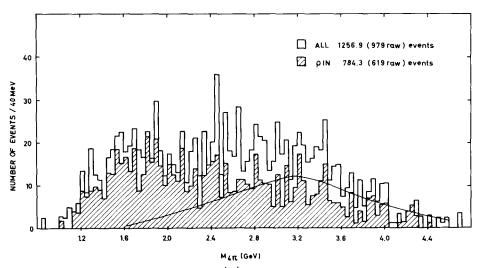


Fig. 2. Four-pion mass distribution in $\gamma p \to \pi^+ \pi^+ \pi^- \pi^- p$ at 9-18 GeV. The shaded spectrum corresponds to events involving at least one ρ . The smooth curve is a peripheral phase space for $\gamma p \to \rho \pi \pi p$ as described in the text.

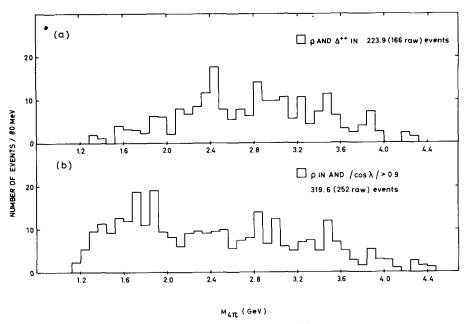


Fig. 3. (a) Four-pion mass distribution of events in the $\rho^0 - \Delta^{++}$ overlap. (b) Four-pion mass distribution of coplanar $\rho \pi \pi$ events.

peaking in the proton-proton momentum transfer (characterized by the exponential slope of $B=4.85\pm0.34~{\rm GeV^{-2}}$ up to $t-t_{\rm min}=-0.6~{\rm GeV^{2}})$ and the experimental bremsstrahlung distribution for the incident photon energy.

For the subsequent analysis we take as ρ' events all $\rho\pi\pi$ events with the mass below 2 GeV*. For these events there is little interference from Δ^{++} since the latter is mostly associated with higher four pion mass as seen in fig. 3a, which is the four pion mass histogram of events in the $\rho^{o} - \Delta^{++}$ overlap (where the Δ^{++} band is defined by $1.16 < m(p \pi^{+}) < 1.32$ GeV).

As shown in fig. 4 the differential cross section for the production of ρ' is strongly peaked forward and is well characterized by an exponential of the form below in practically the entire data region** $0.02 < -t + t_{\min} < 0.6 \text{ GeV}^2$,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t}(\gamma \,\mathrm{p} \to \rho' \,\mathrm{p}) = \frac{\mathrm{d}\sigma}{\mathrm{d}t}\bigg|_{t_{\mathrm{min}}} \exp\left[B(t-t_{\mathrm{min}})\right],$$

where t is the photon to ρ' momentum transfer and t_{\min} its kinematic minimum.

^{*} We allow a generous cut for ρ' events. Because of the severe coplanarity cut subsequently applied we expect the effects of background and other processes to be negligible.

^{**} The data point below 0.02 GeV² has been excluded because of the possible scanning bias as discussed in ref. [7].

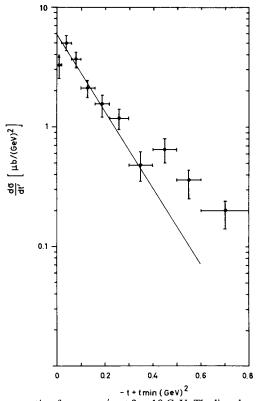


Fig. 4. Differential cross section for $\gamma p \rightarrow \rho' p$ at 9-18 GeV. The line shows the exponential fit as described in the text.

The data yield

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t}\Big|_{t_{\min}} = 6.05 \pm 0.75 \frac{\mu \mathrm{b}}{\mathrm{GeV}^2} ,$$

$$B = 7.56 \pm 0.70 \,\mathrm{GeV}^{-2}$$
.

The slope B is similar to that of the ρ production in the same energy range [7] and essentially all ρ' data have momentum transfer below 0.6 GeV² so that the assumption of the s-channel helicity conservation also in ρ' production appears reasonable.

In fig. 5 we show the $|\cos \lambda|$ distribution for ρ' events where λ is the angle between the planes containing two negative and positive pions respectively $(\cos \lambda = (e_3 \times e_1) \cdot (e_4 \times e_2))$ divided by norms in the same notations as earlier). It is clear that the ρ' decay is dominantly coplanar. To apply the test proposed in our formalism we select events with $|\cos \lambda|$ greater than 0.9 and examine the $\cos \beta$ distribution of these events (amounting to 46% of the total). The corresponding 4π

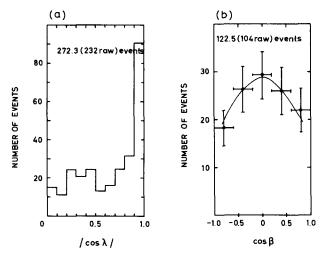


Fig. 5. (a) Distribution of absolute value $\cos \lambda$ for ρ' decay, where λ is the angle between the planes containing two positive and negative pions respectively. (b) $\cos \beta$ distribution for coplanar ρ' decay. The curve shows the fit of the expected form for 1^- .

mass spectrum is shown in fig. 3b. As seen in fig. 5b the $\cos \beta$ distribution exhibits a characteristic $\sin^2 \beta$ term so that in particular the spin is not zero. We obtain a good fit of eq. (13a) for j = 1 with the following value of the decay parameters*

$$a_0 = 0.58 \pm 0.15$$
.

As discussed earlier the presence of a_0 (or the $\sin^2\beta$ term) implies that the spin-parity cannot be 1⁺. The data does not warrant higher spins but if we insist then we get for $i = 2^{**}$

$$a_0 = 0.23 \pm 0.13$$
, $a_1 = -0.06 \pm 0.07$ (must be 0).

Again non-vanishing a_0 rules out 2^- . The choice j = 3 turns out to be bad, because it gives rise to unphysical values for these parameters, namely

$$a_0 = -0.79 \pm 0.33$$
, $a_1 = -0.20 \pm 0.16$, $a_2 = 1.27 \pm 0.47$ (should not exceed $\frac{1}{2}$).

Thus taking advantage of the preferentially complanar decay of ρ^i into $\rho\pi\pi$ and making the consistent assumption of the s-channel helicity conservation we obtain from our data strong evidence that ρ' is a vector meson. This result supports the findings of other experiments. In the e⁺e⁻ annihilation [3] the spin-parity assignment

^{*} In fitting eq. (13) to the $\cos \beta$ distribution the last decay parameter a_j was constrained to satisfy eq. (15). For j greater than one we, of course, used finer binning than shown in fig. 5b.

^{**} For instance, as seen in eq. (13b), a conspicuous dip at x = 0 (or large a_0) would have established the spin-parity to be 2^+ .

of 1⁻ for ρ' follows, if one assumes one-photon exchange, and in the γp experiment [4] utilizing the linear polarization of γ the same conclusion is reached, if one assumes that ρ' is produced conserving the s-channel helicity (as was done in our analysis) and in addition the final state $\rho\pi\pi$ is dominated by the two-body state $\rho\sigma$, were σ is an s-wave isoscalar $\pi\pi$ state.

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