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Study of the $\bar{p}p \to 2\pi^+2\pi^-$ annihilation from S states

OBELIX Collaboration

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Abstract

The results of the spin-parity analysis of $\bar{p}p \to 2\pi^+2\pi^-$ annihilations at very low momentum $\bar{p}$ (≈ 50 MeV/c) are reported. To describe the data the production of the ρ, f_2, a_2 and a_1 mesons and the presence of the $\pi\pi$ interaction in S-wave (the σ term) in the final state are necessary. The best fit solution requires also the presence of a ρ' state of mass and width $M = 1.282 \pm 0.037$, $\Gamma = 0.236 \pm 0.036$ GeV/c^2 and of a heavy pion $\pi(1300)$ of mass and width $M = 1.275 \pm 0.015$, $\Gamma = 0.218 \pm 0.100$ GeV/c^2. The measured fraction of the annihilation cross section into $2\pi^+2\pi^-$ is $(7.61 \pm 0.35) \cdot 10^{-2}$.

In this letter we present the spin-parity analysis of a sample of 31,157 annihilation events $\bar{p}p \to 2\pi^+2\pi^-$ taken in flight at 50 MeV/c momentum $\bar{p}$. This very low incident momentum has been obtained from the 100 MeV/c $\bar{p}$ primary beam of the CERN Low Energy Accelerator Ring (LEAR). The full description of the experimental layout and of the techniques used for the measurements at low momentum are well described in [1], while details on the data selection can be found in [2]. Starting from a total of about 3,800,000 events in flight, we extracted 31,157 events accepted by the 4C kinematical fit to the $\pi^+\pi^-\pi^+\pi^-$ hypothesis at the 5% confidence level. By Monte Carlo it is possible to evaluate a background contribution less than 1% due to the $\pi^+\pi^-\pi^+\pi^-\pi^0$ reaction.

The only existing data comparable with the present ones are the 6,665 $\bar{p}p \to 2\pi^+2\pi^-$ annihilations at rest of Diaz et al. [3], taken about 30 years ago with a liquid hydrogen bubble chamber. In that work the spin-parity analysis was performed by assuming only $^1S_0$ and $^3S_1$ $\bar{p}p$ protonium initial states. However, from the ensemble of the existing data on $\bar{p}p$ annihilations at rest it turns out that the percentage of protonium annihilations in P-waves in liquid is larger than 10% (see [4] and references quoted therein).

The annihilation at a $\bar{p}$ momentum of 50 MeV/c, instead, comes mainly from S-wave, with a P-wave contribution less than $\approx 4\%$ (see [1,5,6] and references quoted therein). Moreover it is easy to show that, if one neglects the P-wave, the singlet and triplet S-wave states do not interfere, so that the spin-parity analysis of low momentum $\bar{p}$ annihilation follows the same formalism used for the protonium annihilations [7].

The value of the $\bar{p}p \to 2\pi^+2\pi^-$ partial cross section at (52.9 ± 2.8) MeV/c $\bar{p}$ momentum is calculated using Eq. (1) of [1], giving a value of $\sigma_{\text{ann}} = (62.7 \pm 2.7)$ mb. Dividing this value for the total $\bar{p}p$ annihilation cross section measured at the same incident momentum and reported in [1], we get a ratio (annihilation frequency in flight for the $2\pi^+2\pi^-$ channel) of

$$f(52.9\,\text{MeV}/c) = (7.61 \pm 0.35) \cdot 10^{-2}. \qquad (1)$$

In this ratio between cross sections, only the statistical errors are relevant, since the systematic ones are mainly due to uncertainties in the beam counting and factorize away.

Annihilations at 50 MeV/c are expected to occur from a statistical mixture of singlet and triplet S-states [1,5]. The measured $2\pi^+2\pi^-$ annihilation frequency can be compared with the same quantity measured at rest in LH_2 by Diaz et al. [3], where S-wave annihilations should dominate: $(f(LH_2) = 6.9 \pm 0.6 \cdot 10^{-2})$. The two values are rather similar. If one neglects P-wave annihilations, this observation could support the conclusion, obtained in [4], that the ratio between spin triplet and spin singlet annihilations is essentially independent of the target density and is very close to the spin statistical ratio 3:1.

The annihilation into four charged pions offers the possibility of studying the production of a system $(\pi^+\pi^-)(\pi^+\pi^-)$ with two dipions in relative S and P waves.

The $\pi\pi$ S-wave, known as the σ meson or the σ interaction, has been studied extensively in the K-matrix formalism in a well known work of Au, Morgan and Pennington (AMP) [8], and is periodically revisited in the light of the new experimental data. An updated reanalysis of the topic can be found in [9], where a full set of reference to earlier works is also given. At present, as can be found also in the last PDG review [10], the isoscalar $\pi\pi$ S-wave is parametrized by the coherent sum of four poles: $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and a pole $f_0(400-1200)$ of uncertain mass and very large width. We recall also that in the earlier work of Diaz et al. [3] a reasonable fit was obtained using a $\pi\pi$ S-wave with poles at 0.8 and 1.1 GeV$/c^2$ and a zero at 0.94 GeV$/c^2$.

For what concerns the $(\pi^+\pi^-)$ P-wave, there is much uncertainty about the existence of the radial excitations of the $\rho(770)$ resonance. Whereas the existence of the $\rho(1700)$ as the ^3D $\bar{q}q$ state seems well established, many analyses found another ρ state with mass around 1450 MeV$/c^2$ [10]. However, also the existence of a $\rho(1260)$ state has been reported [11,12]. In [10] this state is mentioned as controversial, needing confirmation. Some analysis of the pion and $\pi^0\omega$ form factors also lead to the existence of a 1^{--} state around $(1.1-1.3)$ GeV$/c^2$ [13,14]. Recently, a reanalysis of the e^+e^- annihilation in two and four pions [15] pointed to a ρ state with a mass about 1.25 GeV$/c^2$ and a width of 0.35 GeV$/c^2$.

These open problems motivated our efforts for obtaining a sample of annihilations into four pions coming from an almost pure mixture of S-wave initial states, to make the spin-parity analysis easier.

We analyze our data in terms of the isobar model, assuming the following quasi two-body decays:

$$\bar{p}p \to [AB]_L \to [(\pi\pi)_{l_1}(\pi\pi)_{l_2}]_L \tag{2}$$

$$\to [A\pi]_L \to \{[B\pi]_{l_1}\pi\}_L$$

$$\to \{[(\pi\pi)_{l_2}\pi]_{l_1}\pi\}_L \tag{3}$$

Table 1
Final states for the $2\pi^+2\pi^-$ channel considered in the fits discussed below. The notations are from Eqs. (2), (3). Columns A,B,C and D indicate the list of amplitudes included in the fits of Table 2. The symbol σ stands for the 0^{++} $(\pi\pi)$ isoscalar state in S-wave and ρ' for a 1^{--} state of high mass

$\bar{p}p$ state	Final state	I	L	l_1	l_2	A	B	C	D
$^1S_0$	$\rho\rho$	0	1	1	1	×	×	×	×
	$a_2(1320)\pi$	0	2	2	1	×	×	×	×
$^3S_1$	$\rho\sigma$	1	0,2	1	0	×	×	×	×
	$\rho'\sigma$	1	0,2	1	0			×	×
	$\rho f_2(1270)$	1	0	1	2	×	×	×	×
	$a_2(1320)\pi$	1	2	2	1	×	×	×	×
	$a_1(1260)\pi$	1	0	0	1		×	×	×
	$[\pi(1300)]_{\sigma\pi}\pi$	1	1	0	0				×
	$[\pi(1300)]_{\rho\pi}\pi$	1	1	1	1				×

where A and B are resonant states and L, l_1, l_2 are the two particle orbital angular momentum quantum numbers.

We consider all the decay channels listed in Table 1. These channels have been chosen by considering the mesons listed in the last PDG review [10], which are in principle observable within the energy range of the $(\pi\pi)$ and $(\pi\pi\pi)$ invariant mass spectra of the $\bar{p}p \to 2\pi^+2\pi^-$ reaction. Relative angular momenta up to $L, l = 2$ have been considered when allowed by phase space.

The partial wave amplitudes corresponding to the channels of Table 1 can be written as:

$$A_{J^{PC}}(\boldsymbol{q}) = \sum_{\pi}\left[\sum_{Ik} c_I a_k \boldsymbol{Z}_{Jk}(\boldsymbol{q}) F_k(\boldsymbol{q})\right] \tag{4}$$

The index J^{PC} indicates the two $\bar{p}p$ initial states, the sum Σ_π is over all the permutations among identical pions, c_I are the isospin Clebsch-Gordan coefficients, the index k is over all the possible final states labelled by the numbers (L, l_1, l_2), a_k are complex coefficients, $\boldsymbol{Z}$ are the spin functions, F_k describes the energy behaviour of the decay chain and $\boldsymbol{q}$ denotes the set of break-up momenta of the intermediate states of the reaction.

The spin functions $\boldsymbol{Z}$ are written in terms of the covariant Zemach tensors (following [16]), extending the formalism to the case of the $(\rho\rho)_l$ system in a relative angular momentum $l=1$. All these tensors give the same spin amplitudes of the covariant helicity formalism of Chung [17].

The complex functions F_k are in most cases written as a product of Breit-Wigner functions describing the resonances present in the decay chain, parametrized as in [18,19]. However we drop the factor q^L from the centrifugal barrier term multiplying the Breit-Wigner of the K-matrix functions, because it is already present in the Zemach tensor, whereas we maintain this term in the definition of the widths $\Gamma(m)$.

The break-up momenta $q(m)$ for the $\rho \to \pi\pi$ and $f_2 \to \pi\pi$ decays are calculated as $q(m) = \sqrt{1-(4m_\pi^2/m^2)}$ and the two body phase space is given by $\rho(m) = 2q(m)/m$. For the decays of $a_1, a_2 \to \rho\pi$ we take into account the effects due to the decay into an unstable particle by defining the break up momentum $q(m) = m\rho_a(m)/2$ through the generalized phase space integral [20]:

$$\rho_a(m) = \int_{4m_\pi^2}^{(m-2m_\pi)^2} \mathrm{d}m_\rho^2\, \rho_{(m,m_\rho)}\, \rho_{(m_\rho)} |BW_\rho(m_\rho)|^2$$

where

$$\rho_{(m,m_\rho)} = \sqrt{\left[1-\left(\frac{m_\rho+m_\pi}{m}\right)^2\right]\left[1-\left(\frac{m_\rho-m_\pi}{m}\right)^2\right]}$$

and $BW_\rho(m)$ is the Breit-Wigner function for the ρ meson normalized over the available phase space. The function $\rho_a(m)$ obtained in this way is practically identical to that used by Bowler [21] in the study of the a_1 decay.

The function F_k of Eq. (4), in the case of the $\pi\pi$ S-wave interaction (the σ term), is parametrized in terms of the K-matrix formalism [18]. Following the N/D method, it is possible to write the σ interaction in the form [22]:

$$F_{\pi\pi} = \frac{(\Lambda_1+\Lambda_2 s)K_{11} + i\rho_2\Lambda_3(K_{11}K_{22}-K_{12}^2)}{1-i\rho_1K_{11}-i\rho_2K_{22}-\rho_1\rho_2(K_{11}K_{22}-K_{12}^2)} \quad (5)$$

where s is the invariant mass squared of the $\pi\pi$ system, and ρ_1 and ρ_2 are the two-body phase space factors for the decay into $\pi\pi$ and $K\bar{K}$.

The denominator has the poles of the $\pi\pi$ amplitude, whereas the numerator contains the complex Λ parameters which take into account the effects due to the $p\bar{p}$ production process, coupled directly to a $\pi\pi$ intermediate state (the term with Λ_1 and Λ_2) or also through a $K\bar{K}$ intermediate state (the term with Λ_3). In our fits the Λ parameters are left free. The N/D method is physically equivalent to the P-vector approach [18] and the two parametrizations give consistent results [23].

Finally, we treat the overlap of the ρ and ρ' resonances, which have the same spin-parity and the same partial waves, by writing the function F_k of Eq. (4) as a 1×1 K-matrix [18]:

$$F_{\rho\rho'}(m) = \frac{P_{\rho\rho'}}{1-i\rho(m)K_{\rho\rho'}}, \quad (6)$$

where

$$K_{\rho\rho'} = \sum_{i=1}^{2} \frac{m_i\Gamma_i}{\rho(m_i)} B_1^2(q,q_i) \frac{1}{m_i^2-m^2}$$

$$P_{\rho\rho'} = \sum_{i=1}^{2} \beta_i \sqrt{\frac{m_i\Gamma_i}{\rho(m_i)}}\, B_1(q,q_i)\left(\frac{q_i}{q}\right)\frac{1}{m_i^2-m^2}$$

and $m_i, \Gamma_i, \rho(m_i)$ and q_i are mass, width, two-body phase space factor and break-up momentum of the two ρ's. The centrifugal factor B_1 is defined in [17]. We drop also in this case the factor q from the P-vector, because it is already present in the Zemach tensor. The parameters β_i are left free during the fit.

We write the decay intensity as an incoherent sum of the two S-states amplitudes of Eq. (4):

$$W(q) = a^1{}_{S_0}|A^1{}_{S_0}(q)|^2 + (1-a^1{}_{S_0})|A^3{}_{S_1}(q)|^2 \quad (7)$$

where $a^1{}_{S_0}$ and $(1-a^1{}_{S_0})$ are the fractional contributions of these states $(0 \le a^1{}_{S_0} \le 1)$.

The intensity of Eq. (7) has been fitted to the data minimizing event by event the function $-2\ln L$, where L is the likelihood function:

$$L = \prod_{i=1}^{N_{\exp}} W_i \bigg/ \int W \,\mathrm{d}\Omega. \quad (8)$$

In the formula $N_{\exp}$ is the total number of in flight events, W_i is the event by event amplitude and the integral is evaluated over a Monte Carlo sample.

The minimization is performed using the MINUIT package [24]. The Monte Carlo sample is at least three times larger than the measured data sample; it is obtained with the GEANT package [25], taking into account our apparatus acceptance and all the cuts applied to the real data during the analysis. The chosen Monte Carlo statistics is the best compromise between accuracy and computing time. Indeed, in some cases we repeated the fit using a Monte Carlo sample about six times larger than the measured data sample: the best fit parameters so obtained are fully compatible with the ones we get with the smaller MC data sample.

Comparing the fits we refer to the quantity Δ, defined as the difference in $-2\ln L$ (see Table 2). To have an help in choosing the physical hypothesis which reproduces better the data, we also look to a set of ten one-dimensional projections of the five-dimensional phase space. As an intuitive indication on the goodness of the fits, in Table 2 we report a pseudo-χ^2 per degree of freedom (χ^2_{DF}) defined as the sum of the chi squares of each projection: we note, however, that the value of this pseudo-χ^2 strictly depends on the chosen histogram set. The optimum pseudo-χ^2, evaluated on Monte Carlo data, analyzed with the same amplitude used in the simulation, is $\chi^2_{DF} \approx 0.94$.

Inspired by the earlier bubble chamber result [3] we began the fit to the data with the simplest amplitude, obtained by dropping the a_1, ρ' and $\pi(1300)$ contributions from the list of Table 1 (see columns A of Table 2). Since the main contribution to the amplitude comes from the $\rho\sigma$ production, we investigated at first the dependence of the fit results on the form of the assumed $(\pi\pi)$ S-wave amplitude.

In Table 2 it is possible to compare the results relative to a fit performed using the "K_1" $(\pi\pi)$ S-wave amplitude of Au, Morgan and Pennington (AMP) [8] (fit A_1 and Fig. 1a) with those of a fit based on the amplitude of Amsler et al. [26] (fit A_2 and Fig. 1b). The fit of Fig. 1a fails completely to reproduce the deep around 1. GeV/c^2, which is on the contrary better reproduced in Fig. 1b.

The AMP amplitude fits the meson-meson scattering data and contains three narrow poles, a large pole around the $\bar{K}K$ threshold and a pole at about 1.5 GeV/c^2. The amplitude of Amsler et al. [26] is obtained by including in the fit also the recent Crystal Barrel $\bar{p}p \to \pi^0\pi^0\pi^0$ annihilation data taken at LEAR. We obtained results equivalent to those of fit A_2 also with the amplitudes of Anisovich et al. [22] and with the K-matrix solution obtained by us

Table 2
Results for the $\bar{p}p \to 2\pi^+2\pi^-$ fit and percentages of different final states. The percentages are normalized to 100 for each of the two $\bar{p}p$ initial states, whose relative weight is also given. Fit A_1: σ from AMP [8] and amplitudes of column A of Table 1. Fit A_2: σ from [26] and amplitudes of column A of Table 1. Fit B_1: σ from [26] and amplitudes of column B of Table 1. Fit B_2: σ from [26] and amplitudes of column B of Table 1, ρ-ω mixing for the $\rho\rho$ and $\rho\sigma$ final states and $\rho\sigma$ production in $L=2$ angular momentum. Fit C_1: as fit B_2, introducing a ρ' state with M,Γ free. Fit C_2: as fit C_1, introducing a ρ' state with M,Γ fixed to values of Particle Data Group [10]. Fit C_3: as fit C_1, using σ from [33]. Fit D: σ from [33] and amplitudes from column D of Table 1. For brevity only the errors of the better fits are reported

$\bar{p}p$	Final	A_1	A_2	B_1	B_2	C_1	C_2	C_3	D
$^1S_0$	$\rho\rho$	22.5	44.3	43.8	43.2	39.2 ± 1.5	38.2 ± 1.4	41.1 ± 1.5	42.0 ± 1.6
	$a_2\pi$	77.5	55.7	56.2	56.8	60.8 ± 1.5	61.8 ± 1.4	58.9 ± 1.5	58.0 ± 1.6
	% weight	22.5	22.2	21.8	20.0	20.6 ± 0.5	19.7 ± 0.4	20.3 ± 0.5	19.9 ± 0.5
$^3S_1$	$\rho\sigma$	66.0	92.8	86.7	91.0	53.2 ± 0.8	58.6 ± 0.7	52.4 ± 0.7	49.6 ± 0.8
	$\rho'\sigma$	–	–	–	–	14.1 ± 0.2	9.5 ± 0.1	16.5 ± 0.2	15.6 ± 0.3
	ρf_2	32.5	6.6	7.6	7.3	19.1 ± 0.8	21.9 ± 0.7	19.0 ± 0.8	19.7 ± 0.8
	$a_2\pi$	1.5	0.5	0.3	–	–	–	–	–
	$a_1\pi$	–	–	5.5	1.6	13.6 ± 0.8	10.0 ± 0.7	12.1 ± 0.8	13.6 ± 0.9
	$\pi^*\pi$	–	–	–	–	–	–	–	1.4 ± 0.7
	% weight	77.5	77.8	78.2	80.0	79.4 ± 0.5	80.3 ± 0.4	79.7 ± 0.5	80.1 ± 0.5
$-2\ln L$		18726	21624	21920	22321	23332	23112	23463	23580
pseudo-χ^2		4.6	2.4	2.2	2.0	1.8	1.9	1.8	1.7

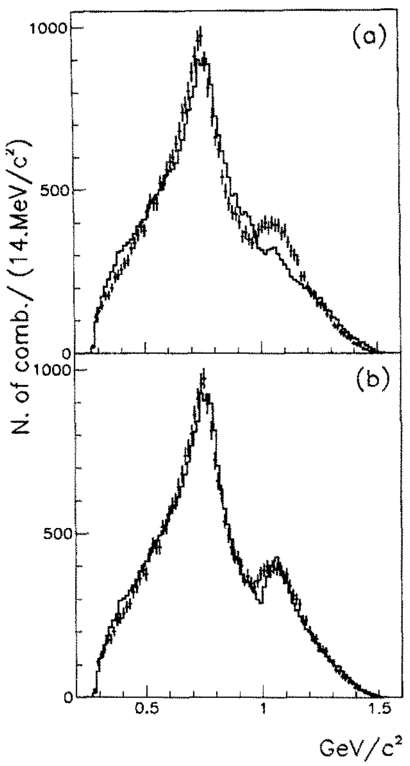

Fig. 1. $\pi^+\pi^-$ invariant mass: comparison between experimental data (points with error bars) and theoretical amplitude (histogram). (a) Results of fit A_1, (b) Results of fit A_2 (see also Table 2).

by fitting our $\bar{p}p \to \pi^+\pi^-\pi^0$ LEAR data [27]. The common characteristic of these amplitudes is the presence of three poles corresponding to the $f_0(980)$, $f_0(1300)$ and $f_0(1500)$ resonances.

The fit of Fig. 1b is still not satisfactory for what concerns the shape of the $\rho^0(770)$ peak and of the $\pi^+\pi^-\pi^\pm$ invariant mass spectrum. We decided then to include in the fit the a_1 production (list B of Table 1), which is seen in the $\pi^+\pi^-\pi^\pm$ invariant mass spectrum as a large shoulder around 1.2 GeV$/c^2$ on the left of the $a_2(1320)$ peak (see Figs. 2b and 2d). The a_1 parameters have been fixed at the PDG values [10]. Although the $a_1(1260)$ state has never been observed in annihilation, its inclusion slightly improves our fit ($\Delta = 296$, between fit A_2 and B_1 in Table 2).

The fit of the ρ^0 peak in Fig. 1b can still be improved taking into account the ρ-ω interference and the $\rho\sigma$ production in $L = 2$ angular momentum.

The ρ-ω interference arises from the G-parity violating electromagnetic mixing of the ω^0 and ρ^0 states [28–30]. This effect has been introduced as a modified ρ-ω Breit-Wigner function parametrized as in [27,31,32]. The fit assigns to the decay ratio $(\omega\pi^+\pi^- \to 2\pi^+2\pi^-)/(\rho\pi^+\pi^- \to 2\pi^+2\pi^-)$ a value of (0.22 ± 0.05) and the likelihood shows only a small improvement ($\Delta = 112$) with respect of the fit B_1 of Table 2.

On the contrary, the $\rho\sigma$ production in $L = 2$ gives a sensible improvement to the fit (fit B_2 of Table 2 has $\Delta = 394$ with respect to fit B_1), but it does not significantly improve the $\pi^+\pi^-$ invariant mass distribution with respect to that shown in Fig. 1b.

At this point we included in the fit the ρ' production associated with σ: due to the available phase space, this is the only channel where the ρ' production is supposed to be relevant.

An acceptable fit is obtained using the $(\pi^+\pi^-)$ S-wave of [26] and all the amplitudes reported in column C of Table 1. The mass and width of the ρ' state are left free, as well as the complex production parameters of Eq. (7): the results of the fit are summarized in Tables 2 and 3, solution C_1. This fit gives an important improvement in reproducing the $\pi^+\pi^-$ invariant mass spectrum (Fig. 2a), while the shape of the 3π invariant mass spectrum (Fig. 2b) is less satisfactory.

In Table 2, column C_2 we reported the results of a fit with a $\rho(1450)$ state with mass and width fixed to the PDG values [10], i.e. $M_{\rho'} = (1465 \pm 25)$ MeV$/c^2$ and $\Gamma_{\rho'} = (310 \pm 60)$ MeV$/c^2$. The solution C_2 is still preferred to the one without ρ', but it is significantly worse than solution C_1.

To study a possible dependence of the ρ' mass on the form of the assumed σ interaction, we repeated fit C_1 with different parametrization of the $(\pi\pi)$ S-wave: the 4-poles K-matrix of [33] (see fit C_3 of Table 2) and the recent parametrization of Bugg et al. [9], consisting in a sum of a K-matrix (reproducing a broad background and the $f_0(980)$ as through the interference between two narrow poles) plus a Breit-Wigner for the $f_0(1300)$ resonance. In our case the presence of the $f_0(1500)$ is irrelevant, due to the available phase space.

Fit C_3 of Table 2 still gives a ρ' state with mass under 1.300 GeV$/c^2$ (see Table 3). Using a $(\pi\pi)$

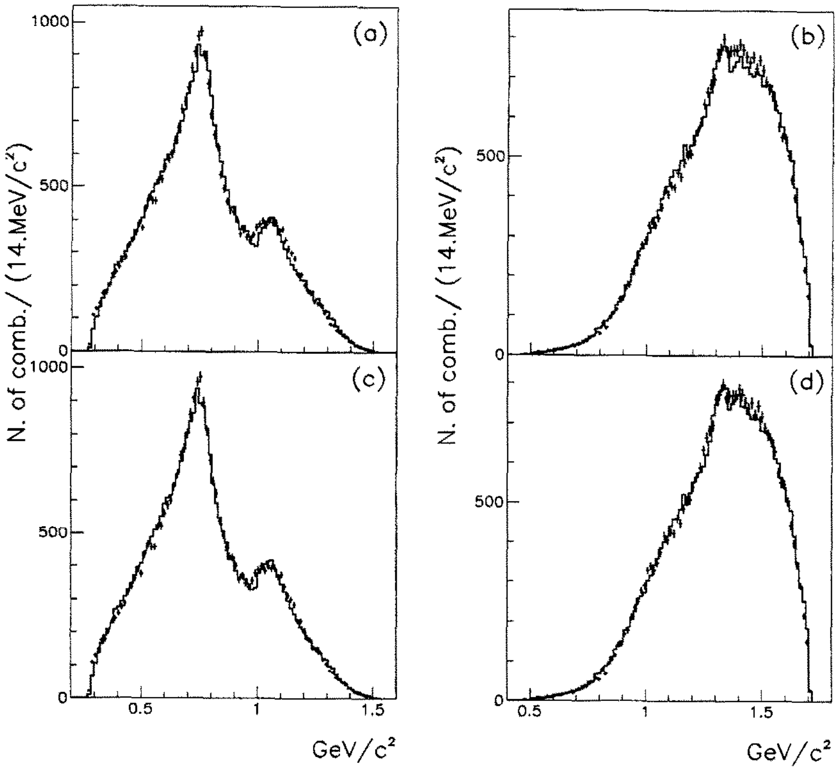

Fig. 2. Comparison between experimental data (points with error bars) and theoretical distribution (histogram). (a) $\pi^+\pi^-$ invariant mass, fit C_1 of Table 2, (b) $\pi^+\pi^-\pi^\pm$ invariant mass fit C_1, (c) $\pi^+\pi^-$ invariant mass, fit D of Table 2, (d) $\pi^+\pi^-\pi^\pm$ invariant mass, fit D.

S-wave from [9] we obtained a fit equivalent to the previous one ($-2\ln L = 23445$), giving a ρ' with mass $M_{\rho'} = 1.276 \pm 0.023$ GeV$/c^2$ and $\Gamma_{\rho'} = 0.251 \pm 0.048$ GeV$/c^2$. The introduction of a second decay channel for the ρ' doesn't change the results of the fit (with a total ρ width of $\Gamma = 0.218$ GeV$/c^2$ we find a width in $\rho\sigma$ of $\Gamma_{\rho\sigma} = 0.018$ GeV$/c^2$). Moreover, the fit rejects the introduction of a third 1^{--} state.

As a further attempt to improve our fits we now include into the decay chain the $\pi(1300)$ state (see Table 1). Here we observe a significant improvement of the fit, mainly in the $\pi^+\pi^-\pi^\pm$ invariant mass distribution. The results are shown in Table 2, fit D and in Figs. 2c, 2d. The mass of the ρ' state is still under 1.300 GeV$/c^2$ (see Table 3) whereas the mass and width of the $\pi(1300)$ are

$$M_{\pi^*} = 1.275 \pm 0.015\,\mathrm{GeV}/c^2$$

$$\Gamma_{\pi^*} = 0.218 \pm 0.100\,\mathrm{GeV}/c^2\,.$$

The ratio between the two decay modes is determined by the fit as $(\pi(1300) \to \sigma\pi)/(\pi(1300) \to \rho\pi) = 5.25 \pm 0.7$. An idea of the overall goodness of fit D is given in Fig. 3.

We remark that a fit with the $\pi(1300)$ state but without ρ' is not satisfying, having $-2\ln L = 22840$ for a π^* state of mass $M = 1.441 \pm 0.023$ and a width $\Gamma = 0.334 \pm 0.096$.

We recall also that the production of the $\pi(1300)$ state has been observed in the $\bar{p}p \to 5\pi^0$ data, with

Table 3
Mass, width in MeV$/c^2$ and relative phase with the $\rho(770)$ of the ρ' state (T-matrix poles) as obtained from fits C_1, C_3 and D of Table 2

	C_1	C_3	D
M_ρ'	1248 ± 23	1274 ± 28	1282 ± 37
Γ_ρ'	242 ± 37	222 ± 27	236 ± 36
$\phi_{\rho\rho'}$	$-(29 \pm 8)°$	$-(28 \pm 8)°$	$-(27 \pm 8)°$

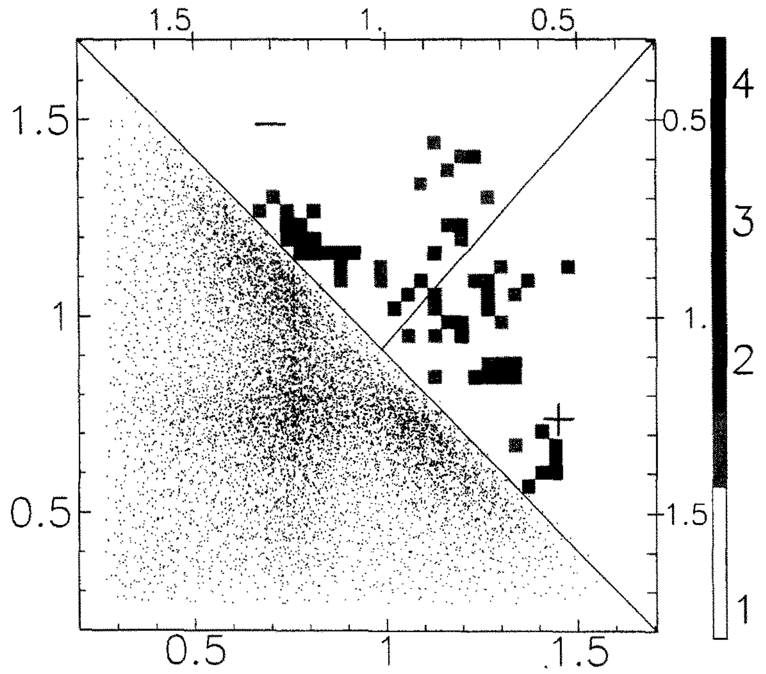

Fig. 3. $M(\pi^+\pi^-)$ vs. $M(\pi^+\pi^-)$. Experimental data are in the low left part of the diagram. The negative and positive differences (in sigma units) between the theoretical (fit *D*) and the experimental scatter plots are in the upper part and in the right part of the plot respectively. The upper left part of the experimental diagram works as a mirror image of the down left part.

mass and width of $M = 1.140$, $\Gamma = 0.340$ GeV/c^2 [36].

We note also that the exotic 1^{-+} isovector state of [37,38] is not accepted by the fit, since the likelihood does not improve, the weight of the state is negligible and its width is reduced practically to zero. The $\pi(1670)$ production is also negligible.

In summary, a satisfactory description of our data is achieved with fit *D* of Table 2. From the annihilation frequency (1) and the percentages reported in that table, the absolute frequency of each decay channel can be obtained. We note, however, that the percentages are calculated neglecting the interference terms.

The characteristics of our best fit solution are: a) a $\pi\pi$ *S*-wave describing both the $\pi\pi$ scattering and the $\bar{p}p \to 3\pi$ annihilation data, parametrized as in (5) to take into account the $\bar{p}p \to 2\pi^+2\pi^-$ production mechanism; b) a non negligible a_1 contribution; c) an a_2 contribution which is relevant from the singlet state and suppressed from the triplet one; d) the presence of a ρ' contribution, of mass and width very stable against the different parametrizations of the σ interaction (see Table 3); e) the observation of the $\pi(1300)$ state decaying to $\sigma\pi$ (mainly) and $\rho\pi$.

The main results of this analysis are probably the presence of the ρ' and $\pi(1300)$ states.

The ρ' signal cannot in principle be identified with the $\rho(1450)$ of PDG, since the mass is lower than the quoted average [10]. Also the value found in the analysis of the $\bar{p}n \to \pi^-\pi^0\pi^0$ annihilation, $M = 1.411 \pm 0.010$ GeV/c^2 [39], is not compatible with our result. However, we note that the ρ' mass value found here is lower, but still compatible ($1.5\,\sigma$) with the mass resulting from our analyses of the $\bar{p}p \to \pi^+\pi^-\pi^0$ and $\bar{n}p \to \pi^+\pi^+\pi^-$ reactions ($M = 1.352 \pm 0.026$ GeV/c^2) [27,35].

For what concerns the $\pi(1300)$ state, our mass and width values agree with those of PDG [10]. We note also that the low production intensity reported in Table 2 ($1.4 \pm 0.7\%$), although small, is not negligible when combined in interference with the other states. However, since this intensity is comparable with that of the *P*-wave initial states, that have been neglected in our analysis, we cannot exclude that other solutions without the $\pi(1300)$ could fit equally well the data. To solve this ambiguity, a combined analysis of these in flight data with data of the same reaction taken at rest from protonium states with liquid and gaseous hydrogen targets is in progress [2].

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