Hybrid and Orbitally Excited Mesons in Full QCD *

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We present results for the hybrid meson spectrum produced by gluonic excitations in full QCD using Wilson fermions. For the spin-exotic mesons with $J^{PC} = 1^{-+}$, 0^{+-} , and 2^{+-} we find the lightest state to be 1^{-+} with a mass of 1.9(2) GeV. Results obtained for orbitally excited mesons are also presented.

1. INTRODUCTION

A quantitative study of the QCD hadronic spectrum should include a study of states with excited orbital angular momentum. These states generally require non-local operators in order to construct observables with the correct symmetries.

Another topic of interest that goes beyond the determination of the ground state spectrum is a study of gluonic mesons: the glueballs and hybrid mesons. Of special interest are the hybrid meson states with J^{PC} quantum numbers that are not allowed in the quark model, the so-called *exotics*. These include the J^{PC} values 1^{-+} , 0^{+-} , and 2^{+-} .

In the quenched theory, the above mentioned topics have been investigated by two separate groups [1,2]. Here we report on an initial study of hybrid and L-excited mesons in two flavour QCD. The aim of the study is to study the effects of dynamical quark effects. A separate study by the MILC Collaboration using improved actions is also underway [2].

2. LATTICE OPERATORS

In order to construct lattice operators with the desired angular momentum or gluonic excitation, we have to combine representations of the 'spin' cubic group (coming from the quark spinors) with the 'orbital' cubic group (coming from the spatial paths). Following [1] we study non-local gluonic fields in specific representations of the *lattice* rotation group.

We work on a $16^3 \times 32$ lattice at $\beta = 5.6$ using 2 degenerate Wilson fermions. We use 4 κ_{sea} values: 0.1560, 0.1565, 0.1570 and 0.1575, which correspond to $m_{\pi}/m_{\rho} = 0.83$, 0.81, 0.76, 0.69 respectively. At each κ_{sea} value we have around 200 independent configurations generated as part of the SESAM Collaboration endeavours. To improve statistics we additionally use a second time source at t = 11.

At each κ_{sea} value we generate a set of local propagators consisting of sources for the quark at (1, 1, 1, t) and at (1, 1, 7, t) for the anti-quark, where t = 1 and 11. At the source the propagators are connected by a path \mathcal{P} consisting of fuzzed gluon links. The path \mathcal{P} is chosen to provide the desired angular momentum (in the case of the *L*-excited mesons) or the gluon excitation (for the hybrid mesons). The resulting hadronic correlations are therefore also by definition gauge invariant. Fuzzed links are used to improve the overlap with the ground state [1].

An alternative procedure to the method used here is to study hybrid mesons using the continuum symmetries [2].

3. L-EXCITED MESONS

L-excited mesons can be studied by choosing \mathcal{P} to be the straight product of fuzzed links connecting the quark and anti-quark at the source and sink. At the source both the direction (here

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Figure 1. The lattice effective mass for the 1^{++} meson vs. time t. For a discussion of the different operators used, see ref. [1].

the \hat{z} or 3 direction) and the length (=6) are fixed. At the sink, on the other hand, we have three possible spatial directions, and the length R can also be varied.

We perform correlated 2-state fits to the effective mass and use as many operators (i.e. different spatial link combinations and choices of R) as possible to constrain the fits as much as possible. As a typical example we show the results for the 1^{++} meson in Fig. 1.

We are also able to determine the hyperfine splitting among the four states in the P-wave multiplet, finding that the singlet state has the lowest mass, while the other three members have masses which are degenerate within the statistical errors. Similar behaviour was observed in the quenched theory [1].

4. HYBRID MESONS

Hybrid mesons are by definition mesons with an excited gluonic component. From studies of static quarks it was found that the lowest lying hybrid states have colour flux from the quark to anti-quark excited in a transverse spatial plane [3]. This can be achieved by the choice of U-shaped paths of fuzzed links at the source and sink. These operators were also successfully used in a study of lighter quark masses in quenched QCD [1].

The lowest lying gluonic excitations have spatial symmetries corresponding to $L^{PC} = 1^{+-}$ and $L^{PC} = 1^{-+}$. Combining these with the $q\bar{q}$ spin representations, we obtain a range of possible J^{PC} values which include the spin-exotic quantum numbers $J^{PC} = 1^{-+}$, 0^{+-} , and 2^{+-} which are not present in the quark model.



Figure 2. The lattice effective mass for the $J^{PC} = 1^{-+}$ exotic hybrid meson vs t. The sources used are U-shaped paths as well as closed colour loops, while at the sink we use only U-shaped paths.

In our simulation the source necessarily again has longitudinal length (r) = 6 fixed, while the transverse length (d) is free. At the sink one can vary both (R and D respectively). From earlier experience we found that the optimum choice of operators has (r, d) = (6, 6) at the source, while at the sink we use (R, D) = (1, 1), (3, 3) and (6, 6). The choice of (6, 6) at both source and sink moreover gives an upper bound on the ground state mass.

The result for the effective mass of the 1^{-+} exotic meson is shown in Fig. 2, while in Fig. 3 we show the extrapolation for the same state to the chiral limit. The value thus obtained, together with those for the other 2 exotic states under consideration here, are listed in Tab. 1. The 1^{-+} ex-

$\operatorname{multiplet}$	J^{PC}	ma (chiral)	mass (GeV)
$\hat{ ho}$	$1^{-+} 3^{-+}$	0.822(100)	1.9(2)
\hat{a}_0	$0^{+-} 4^{+-}$	1.015(250)	2.3(6)
\hat{a}_2	$2^{+-} 4^{+-}$	0.867(500)	2.0(1.1)

Table 1

The (physical) masses of the exotic mesons.

otic is again found to be the lightest state. This result is also found by the MILC investigation [2]. In order to convert the results into physical units we use $a^{-1} = 2.30$ GeV determined from m_{ρ} [4]. These values are given in the last column in Tab. 1.

The estimate for the 1^{-+} state agrees – within the given statistical uncertainty – with the value obtained in the quenched theory. One reason for this might be that the lattice volume (≈ 1.4 fm) is too small – the wave function of hybrid mesons is expected to be large. To investigate these finite volume effects a calculation of the hybrid spectrum on a $24^3 \times 40$ lattice is currently under way.

Taking into account the statistical uncertainty and likely finite volume effects, our result is in fairly good agreement with the 1.6 GeV candidate for the 1^{-+} state proposed in [5].



Figure 3. Extrapolation of the 1⁻⁺ exotic state to the chiral limit. The vertical dashed line is given by $(\kappa_{seq}^{light})^{-1} \simeq 6.31066$ [4].

In Fig. 4 we show the ordering of the hybrid meson levels. The dashed lines represent *L*-excited states. The strong mixing of the hybrid mesons and standard mesons (i.e. with no gluonic excitation) with the same J^{PC} values, already seen in the quenched approximation, is also apparent here. In full QCD there are also other mixing effects: even exotic mesons can now mix with 4q $(q\bar{q}q\bar{q})$ states (e.g. $\pi\eta$). These effects are currently under investigation. We are also planning to implement a third time source to reduce the statistical errors of the results presented above.



Figure 4. Ordering of the hybrid meson levels. The bursts denote the J^{PC} exotic states.

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