Hybrid and Orbitally Excited Mesons in Quenched QCD *

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We use lattice methods to evaluate from first principles the spectrum of hybrid mesons produced by gluonic excitations in quenched QCD with quark masses near the strange quark mass. For the spin-exotic mesons with $J^{PC} = 1^{-+}$, 0^{+-}, and 2^{+-} which are not present in the quark model, we determine the lightest state to be 1^{-+} with mass of 2.0(2) GeV. Results obtained for orbitally excited mesons are also presented.

1. INTRODUCTION

A quantitative study of the hadronic spectrum should include a study of states which are not in the ground state, i.e. those with excited orbital angular momentum. Only then can we get a better insight into the QCD spectrum. However, a study of these states requires non-local operators with the correct symmetries, which can be obtained by either defining explicit P- and/or D-wave operators [1] or by means of standard quark propagators combined with some gluonic flux [2].

Another goal of quantitative studies of QCD is to determine the spectrum of gluonic mesons: the glueballs and hybrid mesons. Of special interest are the hybrid meson states with J^{PC} quantum numbers that are not allowed in the quark model, the so-called *exotics*. These include the J^{PC} values 1^{-+}, 0^{+-}, and 2^{+-}.

2. LATTICE OPERATORS

The group of rotations and inversions of the 3 dimensional spatial lattice is given by the cubic symmetry group O_h (for zero momentum).

In order to construct *lattice* operators with the desired angular momentum or gluonic excitation, we have to combine representations of the 'spin' cubic group (coming from the quark spinors) with the 'orbital' cubic group (coming from the spatial paths). Following [2] we study non-local gluonic fields in specific representations of the lattice rotation group.

In order to construct optimal lattice operators, that is with optimal signal to noise ratio, we need operators with a given O_h representation at the source and/or sink.

The operators used here involve a 'white' source, i.e. a source which, in principle, gets contributions from all quantum numbers, while the sink operators are constructed to project out the desired symmetry [2].

We work on a $16^3 \times 48$ lattice at $\beta = 6.0$ and use a tadpole improved clover action with $\kappa = 0.137$ and $c_{sw} = 1.4785$. This value of the hopping parameter corresponds to a quark mass around the strange quark mass. We have generated a set of 350 local propagators consisting of sources for the quark at (0,0,0,0) and at (0,0,6,0) for the antiquark. At the source the propagators are connected by a path $\mathcal{P}$ consisting of fuzzed gluon links. The path $\mathcal{P}$ is chosen in order to provide the desired angular momentum (in the case of the L-excited mesons) or the gluon excitation (for the hybrid mesons). The resulting hadronic correlations in both cases are therefore also by definition gauge invariant. Fuzzed links are used to improve the overlap with the ground state [2,3].

An alternative procedure to the method used here is to study hybrid mesons using the continuum symmetries [4].
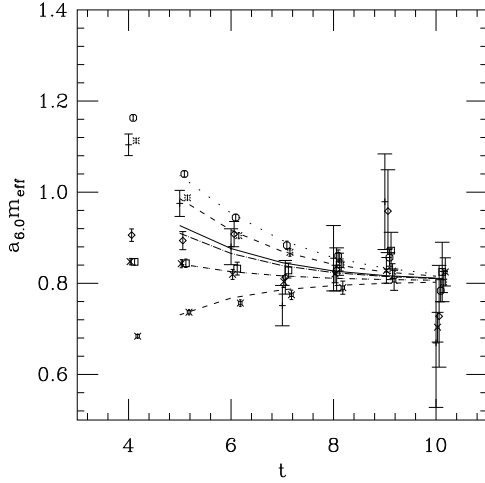
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*Talk presented by P. Lacock

Figure 1. The lattice effective mass for the 1^{++} meson vs. time t. For a discussion of the different operators used, see ref. [2].

3. L-EXCITED MESONS

L-excited mesons can be studied by choosing $\mathcal{P}$ to be the straight product of fuzzed links connecting the quark and anti-quark at the source and sink. At the source both the direction (here the $\hat{z}$ or 3 direction) and the length (=6) are fixed. At the sink, on the other hand, we have three possible spatial directions, and the length R can also be varied.

We perform correlated 2-state fits to the effective mass and use as many operators (i.e. different spatial link combinations and choices of R) to constrain the fits as much as possible. As a typical example, we show the results for the A_1 meson in Fig. 1.

We are also able to determine the hyperfine splitting among the four states in the $P-$wave multiplet, finding that the singlet state has the lowest mass, while the other three members have masses which are degenerate within the statistical errors. This result, reported in [2], is now confirmed using higher statistics.

4. HYBRID MESONS

Hybrid mesons are by definition mesons with an excited gluonic component. From studies of static quarks it was found that the lowest order lying hybrid states have colour flux from the quark to anti-quark excited in a transverse spatial plane [5]. This can be achieved by the choice of U-shaped paths of fuzzed links at the source and sink. Here we use the same construction for lighter quark masses [2].

The lowest lying gluonic excitations have spatial symmetries corresponding to $L^{PC} = 1^{+-}$ and $L^{PC} = 1^{-+}$. Combining these with the $q\bar{q}$ spin representations, we obtain a range of possible J^{PC} values which include the spin-exotic quantum numbers $J^{PC} = 1^{-+}$, 0^{+-}, and 2^{+-} which are not present in the quark model.
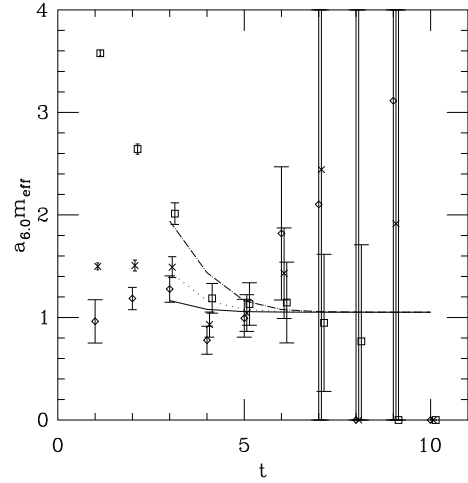

Figure 2. The lattice effective mass for the $J^{PC} = 1^{-+}$ exotic hybrid meson vs t. The source used is a U-shaped path of size 6×6, while the sinks are combinations of U-shaped paths of size 6×6 ($\diamond$), 3×3 ($\square$) and 1×1 (fancy cross).

In our simulation the source necessarily again has longitudinal length (r)=6 fixed, while the transverse length (d) is free. At the sink one can vary both (R and D respectively). From experience we found that the optimum choice of operators has (r,d)=(6,6) at the source, while at the sink we use $(R, D) = (1,1)$, (3,3) and (6,6). The

choice of (6,6) at both source and sink moreover gives an upper bound on the ground state mass.

The results obtained from the correlated 2-state fits are listed for the exotic hybrid mesons in Tab. 1 and shown for the 1^{-+} exotic in Fig. 2 [3]. Our results are consistent with the assumption that the 1^{-+} state is the lightest [6]. Taking account of statistical correlations, we find that the 0^{+-} state is heavier than the 1^{-+} state with a significance of 1 standard deviation.

meson multiplet	J^{PC}	mass Ma	M/M_V	$M_s/M(\phi)$
$\hat{\rho}$	$1^{-+},\ 3^{-+}$	0.95(7)	1.76(13)	1.95(13)
$\hat{a}_0$	$0^{+-},\ 4^{+-}$	1.05(7)	1.94(13)	2.16(13)
$\hat{a}_2$	$2^{+-},\ 4^{+-}$	1.26(13)	2.33(24)	2.62(24)

Table 1
The masses of the exotic hybrid mesons. The J^{PC} values are the lowest two values allowed by the lattice cubic symmetry. The masses are given in lattice units and as a ratio to the vector meson mass m_V. The mass ratio appropriate to $s\bar{s}$ states is given in the last column [3].

In Tab. 1 we also list our lattice results for the hybrid masses as a ratio to the vector meson mass we find on the same lattices, namely $M_V a = 0.54(1)$. Since we only have results at one quark mass value and lattice spacing, we determine the value of the exotic mesons by using the experimental ϕ meson mass to set the scale. Our result for the lightest $s\bar{s}$ hybrid meson then is 1.99(13) GeV. Taking into account some of the possible systematic errors (lattice size effects, extrapolation uncertainties etc.), we round this prediction to 2.0(2) GeV. This value is in agreement with the value obtained by the MILC Collaboration [4].

meson	J^{PC}	mass(Ma)
ρ	1^{--}	0.541(2)
π	0^{-+}	0.413(1)
B	1^{+-}	0.801(20)
A_1	1^{++}	0.808(20)
π_2	2^{-+}	1.165(60)

Table 2
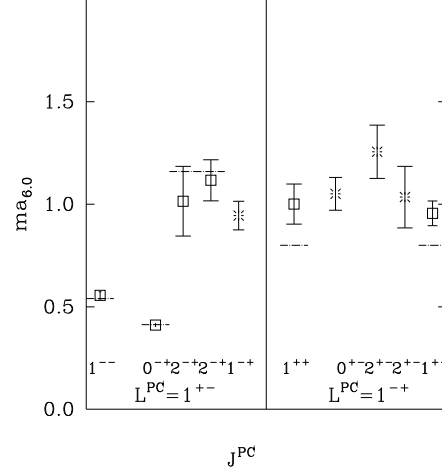The masses of the L-excited mesons shown in Fig. 3.

Figure 3. Ordering of the hybrid meson levels. The bursts denote the J^{PC} exotic states.

In Fig. 3 we show the ordering of the hybrid meson levels. The dashed lines represent L-excited states for the same full set of configurations which are listed in Tab. 2. The strong mixing of the hybrid mesons and standard mesons (i.e. with no gluonic excitation) with the same J^{PC} values even in the quenched approximation is apparent for the pseudoscalar and vector meson cases.

In principle one can also construct hybrid mesons using local gluonic fluxes, i.e. gluon loops starting and ending at the same site. Except for the lowest lying exotic hybrid state, the results obtained are found to be more noisy [3], suggesting that local gluonic operators might not be an ideal choice to provide the gluonic excitations of interest.

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