# Exotic hybrid mesons from improved Kogut-Susskind fermions

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We summarize our measurement of the mass of the exotic  $1^{-+}$  hybrid meson using an improved Kogut-Susskind action. We show results from both quenched and dynamical quark simulations and compare with results from Wilson quarks. Extrapolation of these results to the physical quark mass allows comparison with experimental candidates for the  $1^{-+}$  hybrid meson.

## 1. INTRODUCTION

Experimental evidence suggests the existence of particles with "exotic" quantum numbers  $J^{PC} = 1^{-+}$ , such as the  $\pi_1(1400)$  [1] and the  $\pi_1(1600)$  [2]. Explanations of these include fourquark states and hybrid mesons — mesons with gluonic excitations. Several lattice studies [3–5] have used quenched Wilson and improved Wilson fermions to explore the possibility of such hybrid states. Using both quenched and dynamical Kogut-Susskind quarks, the MILC Collaboration has studied the 1<sup>-+</sup> hybrid closer to the physical quark mass limit.

## 2. 1<sup>-+</sup> HYBRID MESON OPERATOR

We can construct a  $1^{-+}$  hybrid meson operator as the cross product of a color octet rho meson and the chromomagnetic field:  $\rho \times B$  [4]. We have several choices of rho meson operators, but it is convenient to choose the K-S flavor singlet  $\rho_s$ , with the spin  $\otimes$  flavor structure  $\gamma_i \otimes \mathbf{1}$ . This is because each spin component of the  $1^{-+}$  includes two terms, for example:

$$1_x^{-+} = \rho_y B_z - \rho_z B_y.$$
 (1)

If the flavor of the  $\rho$  is dependent on its spin state, the resulting contribution is not a flavor eigenstate.

In computing the field strength, we apply 32APE smearing [6] iterations to the spatial gauge field links. There is a subtlety in using the chromomagnetic field to generate the full  $1^{-+}$  hybrid source operator. We might first determine the magnetic field strength everywhere, then multiply at each site by the quark source vector, and finally symmetrically shift to get the appropriate antiquark source vector. Alternately, we might first apply the symmetric shift to the quark source to form an antiquark source, then multiply by the value of the chromomagnetic field. These methods are equivalent to measuring the B-field at the site of the quark (Fig. 1a) and antiquark (Fig. 1b), respectively. Because the quark and antiquark are spatially separated in the flavor singlet  $\rho_s$ , neither of these represents an eigenstate of charge conjugation. To get an eigenstate of C, we use a symmetrized combination of these for

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Figure 1. Chromomagnetic field measured at the quark (a) and at the antiquark (b).

the  $1^{-+}$  hybrid operator.

## 3. SIMULATION & MEASUREMENT

We measured the connected correlator of the  $1^{-+}$  hybrid state on three sets of  $28^3 \times 96$  lattices generated with the " $a_{tad}^2$ " action [7]. To isolate the effects of dynamical quarks, we used matched quenched and full QCD lattices with  $\beta = 8.40, m_{\rm val}a = 0.016, 0.04,$  for the quenched quarks,  $\beta = 7.18$  for lattices with three degenerate flavors of dynamical sea quarks at the strange quark mass (ma = 0.031) and  $\beta = 7.11$  for lattices with  $m_{u,d} = 0.4m_s$  (ma = 0.0124). These choices of  $\beta$  give approximately the same lattice spacing ( $\sim 0.09$  fm) in the three cases. The corresponding choices of quark mass allow simulation at roughly equivalent values of  $(m_{\rm PS}/m_V)^2$ , the square of the ratio of the psuedoscalar to vector meson masses.

#### 4. RESULTS

We fit the measured correlators to the sum of oscillating and normal exponentials:

$$C(t) = A_1 e^{-M_{1-+}t} + A_2(-1)^t e^{-m_2 t} + A_3(-1)^t e^{-m_3 t}, \qquad (2)$$

where  $M_{1^{-+}}$  is the hybrid meson mass of interest and  $m_2$  and  $m_3$  are masses of nonexotic parity partner states which have oscillating correlators in the Kogut-Susskind formulation. We performed both four and five parameter fits. For the four parameter fits, we fix  $A_3 = m_3 = 0$ . For the five parameter fits we fix  $m_3$  to a pre-determined  $a_1$  meson mass and fit for  $A_3$  as well. We varied the range of the fit and tried to choose values for  $M_{1^{-+}}$  corresponding to high-confidence fits that were insensitive to  $t_{\text{max}}$  and  $t_{\text{min}}$ , the limits of the fit range.



Figure 2. Summary of  $1^{-+}$  hybrid meson mass predictions as a function of  $(m_{\rm PS}/m_V)^2$ . The bold octagon represents the linear extrapolation of  $n_f = 0$  data to  $(m_{\rm PS}/m_V)^2 = 0.033$ .

In Fig. 2 we summarize our results along with the results of previous Wilson quark studies by the MILC group [4] and the UKQCD collaboration [3] as well as recent results from the Zhongshan University group [5] using Wilson quarks on an anisotropic lattice. We use the string tension  $\sigma$  to establish the lattice length scale and plot  $M_{1^{-+}}/\sqrt{\sigma}$ . For comparison, we include the 1<sup>-+</sup> experimental candidates  $\pi_1(1400)$  and  $\pi_1(1600)$  at the physical value of  $(m_{\rm PS}/m_V)^2 =$  $(m_{\pi}/m_{\rho})^2 = 0.033$ . We use  $\sqrt{\sigma} = 440 \pm 38 \text{MeV}$ [8] for the vertical scale.

For the quenched lattices we were able to fit the data with reasonable confidence levels (25-50%) for valence quark masses ma = 0.016 and ma = 0.040. Fig. 3 shows an example of fits for the quenched lattices for  $t_{\text{max}} = 15$ . We performed a linear extrapolation of these results to the physical value of  $(m_{\text{PS}}/m_V)^2$ . (Fig. 2.)

For lattices with three degenerate sea quarks at  $m_s$ , we were also able to extract a value for  $M_{1-+}$  in reasonable agreement with the quenched result. The fits, however, exhibited slightly larger statistical errors, and a slight dependence on range.



Figure 3.  $M_{1^{-+}}$  vs.  $t_{\min}$  for  $\beta = 8.40$  quenched lattices,  $t_{\max} = 15$ , 5-parameter fit. Symbol size is proportional to confidence level.

The lattices with  $m_{u,d} = 0.4m_s$  proved more interesting. In the case of a valence quark mass equal to  $m_s$  (ma = 0.031), we use the masses of the  $s\bar{s}$  pseudoscalar state and the vector  $\phi$  state, measured on the same lattices, for  $m_{\rm PS}$  and  $m_V$ respectively. The fitted mass agrees with those of the quenched and three-flavor results within two standard deviations, but with larger systematic errors, estimated from the dependence on fit range.

In the case of the light valence quark (ma = 0.0124), we were unable to say much about the  $1^{-+}$  hybrid mass with any confidence. The fits were very range dependent, indicating the likely presence of four-quark states into which the hybrid can decay.

## 5. DISCUSSION AND CONCLUSIONS

Interpretation of lattice results depends crucially on understanding the lattice length scale. We use the string tension  $\sigma$  as opposed to other static potential scales such as  $r_0$  or  $r_1$  for two reasons. First, hybrid mesons are extended objects that feel the linear part of the static quark potential, where  $\sigma$  is defined, more than the Coulombic part, where  $r_0$  is defined. Second, using  $\sigma$ to define the lattice spacing brings quenched and dynamical quark data into closer agreement for both the hybrids and for low-lying non-exotic hadrons [9]. Note that if we use  $r_1 = 0.34$  fm, our quenched data extrapolates to 2033(70) MeV, whereas using  $\sqrt{\sigma} = 440$  MeV we get 1854(65) MeV, quoting statistical errors only. The  $m_{u,d} = 0.4m_s$  data illustrates that dynamical quarks introduce new and significant processes that contribute to the 1<sup>-+</sup> propagator. Mixing with four-quark states is one possibility. We now have ahead of us the task of understanding these contributions so that we can make useful predictions of the 1<sup>-+</sup> hybrid mass in the presence of dynamical quarks.

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