Hybrid mesons from anisotropic lattice QCD with the clover and improved gauge actions

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We study hybrid mesons from the clover and improved gauge actions at $\beta = 2.6$ on the anisotropic $12^3 \times 36$ lattice using our PC cluster. We estimate the mass of 1^{-+} light quark hybrid as well as the mass of the charmonium hybrid. The improvement of both quark and gluonic actions, first applied to the hybrid mesons, is shown to be more efficient in reducing the lattice spacing and finite volume errors.

1. INTRODUCTION

Lattice QCD is the ideal approach not only for computing $\bar{q}q$ meson spectrum, but also for hybrids and glueballs. However, the lattice technique is not free of systematic errors. The Wilson gauge and quark actions suffer from significant lattice spacing errors, which are smaller only at very large β, and very large lattice volume is required to get rid of finite size effects.

There have been several quenched lattice calculations of hybrid meson masses, part of them are listed in Tab. 1. In Ref. [1], the Wilson gluon action and quark action were used. In Refs. [2,3], the authors used Wilson gauge action and SW improved quark action. For the hybrid mesons containing heavy quarks $\bar{Q}Qg$, the NRQCD action[4] and the LBO action[5] have also been applied. There is also a recent work using the improved KS quark action[6].

In this work, we employ *both improved gluon and quark actions on the anisotropic lattice*, which should have smaller systematic errors, and should be more efficient in reducing the lattice spacing and finite volume effects. We will present data for the 1^{-+} hybrid mass and the splitting between the 1^{-+} hybrid mass and the spin averaged S-wave mass for charmonium. Details can be found in Ref. [7].

2. ACTIONS

The total lattice action is $S = S_g + S_q$. The improved gluonic action S_g is [8,9]:

$$
\begin{aligned}
S_g &= -\beta \frac{1}{\xi} \sum_{x,j<k} \left(\frac{5}{3} \frac{P_{j,k}}{u_s^4} - \frac{1}{12} \frac{R_{j,k}}{u_s^6} - \frac{1}{12} \frac{R_{k,j}}{u_s^6} \right) \\
&- \beta \xi \sum_{x,j} \left(\frac{4}{3} \frac{P_{j,4}}{u_s^2} - \frac{1}{12} \frac{R_{j,4}}{u_s^4} \right), \qquad (1)
\end{aligned}
$$

where P stands for a 1×1 plaquette and R for a 2×1 rectangle. The SW improved action for quarks[10,11] is

$$
\begin{aligned}
S_q &= \sum_x \bar{\psi}(x)\psi(x) \\
&- \kappa_t \sum_x [\bar{\psi}(x)(1-\gamma_0)U_4(x)\psi(x+\hat{4}) \\
&+ \bar{\psi}(x)(1+\gamma_0)U_4^\dagger(x)\psi(x-\hat{4})] \\
&- \kappa_s \sum_{x,j} [\bar{\psi}(x)(1-\gamma_j)U_j(x)\psi(x+\hat{j}) \\
&+ \bar{\psi}(x)(1+\gamma_j)U_j^\dagger(x-\hat{j})\psi(x-\hat{j})] \\
&+ i\kappa_s C_s^{TI} \sum_{x,j<k} \bar{\psi}(x)\sigma_{jk}\hat{F}_{jk}(x)\psi(x) \\
&+ i\kappa_s C_t^{TI} \sum_{x,j} \bar{\psi}(x)\sigma_{j4}\hat{F}_{j4}(x)\psi(x), \qquad (2)
\end{aligned}
$$

where $\hat{F}$ stands for the clover-leaf construction[12] for the gauge field tensor. Tadpole im-

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provement is carried out so that the actions are more continuum-like.

3. SIMULATIONS

On our PC cluster[13–15], the SU(3) pure gauge configurations were generated with the gluon action in Eq. (1) using Cabibbo-Marinari pseudo-heatbath algorithm. The configurations are decorrelated by SU(2) sub-group over-relaxations. We calculated the tadpole parameter u_s self-consistently. 90 independent gauge configurations at $\beta = 2.6$ and $\xi = 3$ on the $12^3 \times 36$ lattice were stored. Although such an ensemble is not very big, it is bigger than earlier simulations by UKQCD and MILC collaborations[1–3] on isotropic lattices.

The quark propagator was obtained by inverting the matrix Δ in $S_q = \sum_{x,y} \bar{\psi}(x)\Delta_{x,y}\psi(y)$ in Eq. (2) by means of BICGStab algorithm. The residue is of $O(10^{-7})$. We computed the correlation functions with various sources and sinks[7], at four values of the Wilson hopping parameter ($\kappa_t = 0.4119$, 0.4199, 0.4279, 0.4359).

In Fig. 1, we plot the effective masses for the π, ρ, f_1 ordinary mesons and 1^{-+} exotic meson at $\kappa_t = 0.4359$. For the ordinary mesons, we used the their corresponding operator as both source and sink. For the exotic meson, we tried two different cases: (1) the 1^{-+} operator as both source and sink; (2) the q^4 source and 1^{-+} sink, which give consistent results within error bars.

The CP-PACS, MILC and UKQCD collaborations used the unimproved Wilson gauge action to generate configurations. They had to work on very large $\beta(> 6)$, corresponding to very small $a_s(< 0.1$ fm), to get rid of the finite spacing errors. They had also to use very large lattices $L^3 \geq 20^3$, to avoid strong lattice size effects at such small a_s. In comparison, our lattices are much coarser ($a_s = 0.33$ fm), and the number of lattice sites is much smaller. Our finite size effects could be ignored, for the physical size of the spatial lattice is $12^3 a_s^3 = (3.96 \text{ fm})^3$ and should be big enough. Our results for the effective mass indicate the existence of a much wider plateau than in the previous work on isotropic lattices.

4. RESULTS

By extrapolating the effective mass of the 1^{-+} hybrid meson to the chiral limit, and using a_t determined from the ρ mass, we get $2013 \pm 26 \pm 71$ MeV. In Tab. 1, we compare the results from various lattice methods. Our result is consistent with the MILC data[1], obtained using the Wilson gluon action and clover quark action on much larger isotropic lattices and much smaller a_s.

We also show our results in Tab. 1 for the 1^{-+} hybrid meson mass in the charm quark sector, using the method discussed in Refs. [1,3]. Our corresponding $\kappa_t^{\text{charm}} = 0.1806(5)(18)$ is obtained by tuning $(m_\pi(\kappa_t \to \kappa_t^{\text{charm}}) + 3m_\rho(\kappa_t \to \kappa_t^{\text{charm}}))/4 = (m_{\eta_c} + 3m_{J/\psi})/4$=3067.6 MeV, where on the right hand side, the experimental inputs $m_{\eta_c} = 2979.8$ MeV and $m_{J/\psi} = 3096.9$ MeV are used. The 1^{-+} hybrid meson mass at our $1/\kappa_t^{\text{charm}}$ is $m_{1^{-+}} = 4369 \pm 37 \pm 99$ MeV, is consistent with the MILC data[1]. The splitting between the hybrid meson mass and the spin averaged S-wave mass $[m_{1^{-+}} - (m_{\eta_c} + 3m_{J/\psi})/4]$, at our κ_t^{charm} is $1302 \pm 37 \pm 99$ MeV, consistent with the CP-PACS data, obtained using the Wilson gluon action and NRQCD quark action on much larger anisotropic lattices and much smaller a_s.

As a byproduct, we give the f_1 P-wave 1^{++} meson in the chiral limit, as well as their experimental values[16]. If we assume that the pion is massive and $f_1(1420)$ is made of $\bar{s}s$, the f_1 P-wave 1^{++} meson mass would be $1499 \pm 28 \pm 65$ MeV.

5. SUMMARY

To summarize, we have used the tadpole-improved gluon action and clover action to compute the hybrid meson masses on much coarser anisotropic lattices. The main results are given in Tab. 1 and compared with other lattice approaches. In our opinion, our approach is more efficient in reducing systematic errors due to finite lattice spacing.

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Light $1^{-+}\bar{q}qg$ (GeV)	Method	Ref.
1.97(9)(30)	Isotropic S_g(W) + S_q(W)	MILC97[1]
1.87(20)	Isotropic S_g^{TI}(W) + S_q^{TI}(SW)	UKQCD97[2]
2.11(10)	Isotropic S_g^{TI}(W) + S_q^{TI}(SW)	MILC99[3]
2.013(26)(71)	Anisotropic $S_g^{\mathrm{TI}}(1\times1+2\times1)+S_q^{\mathrm{TI}}$(SW)	**ZSU (this work)**
$1^{-+}\bar{q}qg$ (GeV)	Method	Ref.
4.390 (80) (200)	Isotropic S_g(W) + S_q(W)	MILC97[1]
4.369 (37) (99)	Anisotropic $S_g^{\mathrm{TI}}(1\times1+2\times1)+S_q^{\mathrm{TI}}$(SW)	**ZSU (this work)**
$1^{-+}\bar{c}cg$ -1S $\bar{c}c$ splitting (GeV)	Method	Ref.
1.34(8)(20)	Isotropic S_g(W) + S_q(W)	MILC97[1]
1.22(15)	Isotropic S_g^{TI}(W) + S_q^{TI}(SW)	MILC99[3]
1.323(13)	Anisotropic S_g^{TI}(W) + S_q^{TI}(NRQCD)	CP-PACS99[4]
1.19	Isotropic $S_g^{\mathrm{TI}}(1\times1+2\times1)+S_q^{\mathrm{TI}}$(LBO)	JKM99[5]
1.302(37)(99)	Anisotropic $S_g^{\mathrm{TI}}(1\times1+2\times1)+S_q^{\mathrm{TI}}$(SW)	**ZSU (this work)**

Table 1
Predictions for the masses of hybrid mesons. Abbreviations: W for Wilson, $1\times1+2\times1$ for the plaquette terms plus the rectangle terms, SW for Sheikholeslami-Wohlert (Clover), TI for tadpole-improved, NRQCD for non-relativistic QCD, and LBO for leading Born-Oppenheimer.

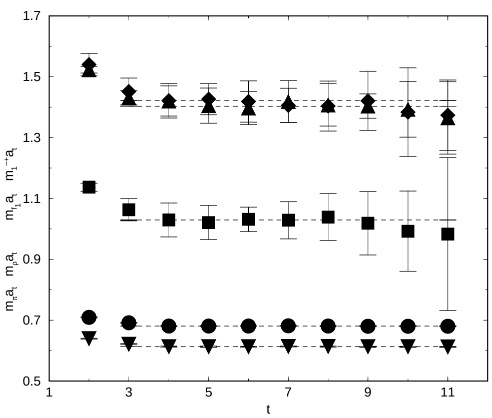

Figure 1. Effective masses for the π (triangle down), ρ (circles), f_1 P-wave (square) mesons and 1^{-+} exotic meson (the diamond for 1^{-+} source and the triangle up for the q^4 source).

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