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Drift Distortions in Alice TPC Field Cage

D. Vranic

Abstract

Calculations of drift distortions for the TPC field cage with two coaxial cylinders with closed ends are presented. The calculation method is based on the solutions of Laplace's equation in cylindrical coordinates. It is applied to the Alice TPC to study a variety of possible defects in design and construction of the field cage. Distortions are calculated due to finite width of the equipotential strips, errors in resistor chain values, ordering of the resistors, shorted strips, mismatch of the field cage cylinder ground end voltage with the pad plane, deformations of the pad plane, etc. The results show that the precision in mechanical construction has to be better than 0.5 mm. The finite width of the equipotential strips is not a problem and a proper ordering of resistors in the resistor chain can significantly reduce errors due to the tolerance of the resistance. A shorted strip pair has large distortion and it would be necessary to be repaired in order to avoid large correction.

1. Introduction

If we assume that the distortion is relatively small, so that the error in axial electrostatic field component (drift direction) can be ignored, then the deviation in the electron path in the radial direction is equal to the following integral (Radial Distortion Integral, RDI) [1],[2]:

$$\Delta\rho(\rho,z) = \int_z^0 \frac{E_\rho(\rho,z)}{E_z} dz \tag{1}$$

where ρ, z are radial and azimuthal coordinates of the electron creation point. It is assumed that E_z=const so that for evaluation of the integral we need to solve Laplace's equation using just the error potential.
For the normal case with a magnetic field parallel to the z axis, the magnitude of the distortion is the same and its direction is a function of the $\omega\tau$ (drift gas mixture).

2. Laplace's Equation in Cylindrical Coordinates

In cylindrical coordinates Laplace's equation takes the form [3]:

$$\frac{\partial^2\Phi}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\Phi}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2} = 0$$

The separation of variables

$$\Phi(\rho,\phi,z) = R(\rho)\,Q(\phi)\,Z(z)$$

leads to the three ordinary differential equations:

$$\frac{d^2Q}{d\phi^2} + m^2 Q = 0 \tag{2a}$$

$$\frac{d^2Z}{dz^2} - k^2 Z = 0 \quad \text{or} \quad \frac{d^2Z}{dz^2} + k^2 Z = 0 \tag{2b}$$

$$\frac{d^2R}{d\rho^2} + \frac{1}{\rho}\frac{dR}{d\rho} + \left(k^2 - \frac{m^2}{\rho^2}\right)R = 0 \quad \text{or} \quad \frac{d^2R}{d\rho^2} + \frac{1}{\rho}\frac{dR}{d\rho} - \left(k^2 + \frac{m^2}{\rho^2}\right)R = 0 \tag{2c}$$

The choice of the sign for the separation constant k^2 depends on the boundary conditions for a given problem.
The solutions of the first two equations are elementary:

$$Q(\phi) = C_1\cos(m\phi) + C_2\sin(m\phi) \tag{3a}$$

$$Z(z) = C_3\exp(kz) + C_4\exp(-kz) \quad \text{or} \quad Z(z) = C_3\cos(kz) + C_4\sin(kz) \tag{3b}$$

In order that the potential be single valued, m must be integer.
For the radial equation the solutions are Bessel functions or modified Bessel functions:

$$R(\rho) = C_5 J_m(k\rho) + C_6 Y_m(k\rho) \quad \text{or} \quad R(\rho) = C_5 I_m(k\rho) + C_6 K_m(k\rho) \tag{3c}$$

For coaxial cylinders with rotationally symmetric boundary conditions m=0:

$$Q(\phi) = C_1\phi + C_2 \qquad (C_1 = 0) \tag{4a}$$

$$Z(z) = C_3 \exp(kz) + C_4 \exp(-kz) \quad \text{or} \quad Z(z) = C_3 \cos(kz) + C_4 \sin(kz) \tag{4b}$$

$$R(\rho) = C_5 J_0(k\rho) + C_6 Y_0(k\rho) \quad \text{or} \quad R(\rho) = C_5 I_0(k\rho) + C_6 K_0(k\rho) \tag{4c}$$

For k^2=0 we have elementary solutions for all three equations:

$$Q(\phi) = C_1 \cos(m\phi) + C_2 \sin(m\phi) \quad \text{or for } m=0 \quad Q(\phi) = C_1 \phi + C_2 \qquad (C_1 = 0) \tag{5a}$$

$$Z(z) = C_3 z + C_4 \tag{5b}$$

$$R(\rho) = C_5 \rho^m + C_6 \rho^{-m} \quad \text{or for } m=0 \quad R(\rho) = C_5 \ln(\rho) + C_6 \tag{5c}$$

If there is no ϕ dependence in the problem we have the simple solution:

$$\Phi(\rho, z) = (C_3 z + C_4)(C_5 \ln(\rho) + C_6) \tag{6}$$

3. Radial Distortion formula

Since the error potential vanishes at the end caps of the field cage, the sine function is the appropriate choice for Fourier expansion with additional condition for the separation constant k^2:

$$k_n = \frac{n\pi}{L} \tag{7}$$

L is the length of the field cage, and n is integer from 1 to infinity. From equations (4) follows expansion for the potential:

$$\Phi(\rho, z) = \sum_{n=1}^{\infty} \left[A_n\, I_0(k_n \rho) + B_n\, K_0(k_n \rho) \right] \sin(k_n z) \tag{8}$$

The coefficients A_n and B_n can be evaluated using the potentials on the boundaries:

$$\tfrac{2}{L} \int_0^L \Phi(\rho_1, z) \sin(k_n z) dz = A_n\, I_0(k_n \rho_1) + B_n\, K_0(k_n \rho_1) \tag{9a}$$

$$\tfrac{2}{L} \int_0^L \Phi(\rho_2, z) \sin(k_n z) dz = A_n\, I_0(k_n \rho_2) + B_n\, K_0(k_n \rho_2) \tag{9b}$$

where ρ_1 and ρ_2 are inner and outer radius of the field cage cylinders. From (8) and using the derivative relations:

$$\frac{d}{dx} I_0(x) = I_1(x) \qquad \frac{d}{dx} K_0(x) = -K_1(x)$$

follows the expression for the radial component of the electric field:

$$E_\rho(\rho, z) = -\frac{d}{d\rho} \Phi(\rho, z) = -\sum_{n=1}^{\infty} k_n \left[A_n\, I_1(k_n \rho) - B_n\, K_1(k_n \rho) \right] \sin(k_n z) \tag{10}$$

From RDI (1) , $E_z = -\dfrac{V}{L}$, and the integral of (10) we get as final result for the radial distortions:

$$\Delta\rho(\rho, z) = L \sum_{n=1}^{\infty} \left[a_n\, I_1(k_n \rho) - b_n\, K_1(k_n \rho) \right] \left(\cos(k_n z) - 1 \right) \tag{11}$$

where:

$$a_n = \frac{A_n}{V} = \frac{1}{D_n} [S2_n\, K_0(k_n \rho_1) - S1_n\, K_0(k_n \rho_2)] \tag{12a}$$

$$b_n = \frac{B_n}{V} = \frac{1}{D_n}[S1_n I_0(k_n \rho_2) - S2_n I_0(k_n \rho_1)] \tag{12b}$$

$$D_n = I_0(k_n \rho_2) K_0(k_n \rho_1) - I_0(k_n \rho_1) K_0(k_n \rho_2) \tag{13}$$

$$S1_n = \frac{1}{V}\frac{2}{L}\int_0^L \Phi(\rho_1, z)\sin(k_n z)dz \tag{14a}$$

$$S2_n = \frac{1}{V}\frac{2}{L}\int_0^L \Phi(\rho_2, z)\sin(k_n z)dz \tag{14b}$$

Since potentials and fields are additive, to evaluate RDI integral (1) we can use just the error potential on the boundary. From Eq. (14) we see that the distortion depends only on 'normalized error potential' (error potential divided by drift voltage V).

The number of terms required in the sum (11) depends on the form of the error potential on the boundary. For each particular case, one can use the expression for the potential

$$\Phi(\rho, z) = V\sum_{n=1}^{\infty}\left[a_n\, I_0(k_n \rho) + b_n\, K_0(k_n \rho)\right]\sin(k_n z) \tag{15}$$

to check the number of terms required for a reasonable reproduction of the error potential.

Integrals in the relations (14) can be evaluated using the fast Fourier transform. That reduces significantly the computing time in the case when it is necessary to sum few hundreds of terms.

Ordinary Bessel functions can be seen as 'distorted' sine and cosine functions. Modified Bessel functions are just Bessel functions of pure imaginary argument and can be seen as 'distorted' exponential functions [4].
For practical purposes it is useful to introduce scaled modified Bessel functions:

$$SI_0(x) = I_0(x)e^{-x} \qquad SK_0(x) = K_0(x)e^{x} \tag{16a}$$

$$SI_1(x) = I_1(x)e^{-x} \qquad SK_1(x) = K_1(x)e^{x} \tag{16b}$$

Except for small x, they are slow varying functions (see Fig 1.). For large x the only dependence is in the exponential part.

The limiting forms for large x are:

$$SI_m(x) \to \frac{1}{\sqrt{2\pi x}} \qquad SK_m(x) \to \sqrt{\frac{\pi}{2x}} \tag{17}$$

Using scaled modified functions we get for the coefficients (12):

$$a_n = \frac{1}{D_n}[S2_n SK_0(k_n \rho_1)e^{-k_n\rho_1} - S1_n SK_0(k_n \rho_2)e^{-k_n\rho_2}]$$

$$b_n = \frac{1}{D_n}[S1_n SI_0(k_n \rho_2)e^{k_n\rho_2} - S2_n SI_0(k_n \rho_1)e^{k_n\rho_1}]$$

$$D_n = SI_0(k_n \rho_2)SK_0(k_n \rho_1)e^{k_n(\rho_2-\rho_1)} - SI_0(k_n \rho_1)SK_0(k_n \rho_2)e^{-k_n(\rho_2-\rho_1)}$$

$$a_n I_1(k_n\rho) = \frac{1}{D_n}[S2_n SK_0(k_n\rho_1)SI_1(k_n\rho)e^{k_n(\rho-\rho_1)} - S1_n SK_0(k_n\rho_2)SI_1(k_n\rho)e^{-k_n(\rho_2-\rho)}] \qquad (18a)$$

$$b_n K_1(k_n\rho) = \frac{1}{D_n}[S1_n SI_0(k_n\rho_2)SK_1(k_n\rho)e^{k_n(\rho_2-\rho)} - S2_n SI_0(k_n\rho_1)SK_1(k_n\rho)e^{-k_n(\rho-\rho_1)}] \qquad (18b)$$

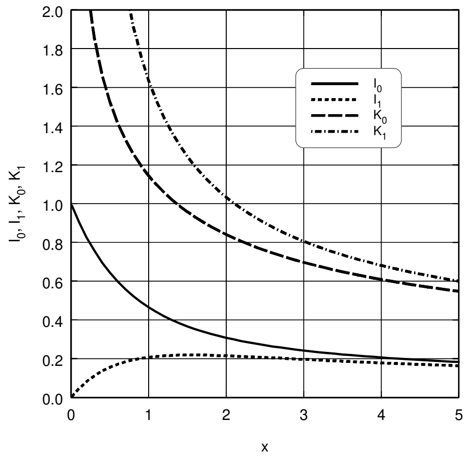

Figure 1 Scaled modified Bessel functions

4. Distortions from Finite Width Equipotential Strips

For the calculation of this distortion we assume that the gap between strips is infinitely small. There are two good reasons for that approximation. First, distortion for finite gap size will be smaller, and second reason is that in that case we can use exact analytical expressions for field lines (assuming large radii of the field cage).

The error potential due to finite width of the strips is equal zero at the center of the gap and at the center of the strip (see Fig 2). It increases linearly with the z distance from the gap. The Fourier transforms (16) for this 'saw teeth" form can be evaluated analytically,

$$S1_n = S2_n = \frac{2}{n\pi}$$

where $n = 2N_s, 4N_s, \ldots$ (N_s is number of strips over full length of the field cage)

In this case we have an error potential with high spatial frequency and large arguments for the modified Bessel functions so that we can use the asymptotic expressions (17). Then relation (11) gets the form

$$\Delta\rho(\rho,z)=\sum_{m=1}^{\infty}\frac{2w}{m\pi}\left\{\sqrt{\frac{\rho_1}{\rho}}\exp\left[-\frac{2\pi m}{w}(\rho-\rho_1)\right]-\sqrt{\frac{\rho_2}{\rho}}\exp\left[-\frac{2\pi m}{w}(\rho_2-\rho)\right]\right\}\left(1-\cos(\frac{2\pi m}{w}z)\right) \tag{19}$$

where ρ_1, ρ_2 are inner and outer cylinder radii and w is strip width

$w=\dfrac{L}{N_s}$ (L is the length of the field cage)

From Eq. (19) it follows that

- the distortion decreases exponentially with the distance from the cage walls
- maximal distortion occurs for the electrons produced at $z=n\dfrac{w}{2}$ (strip center)
- minimal distortion occurs for the electrons produced at $z=n\,w$ (gap center)
- the distortion is such that the electrons are pushed away from the cage walls

From Eq. (19) follows simple expression for distortion close to the cage wall (we take only the first term in summation):

$$distortion=\frac{2w}{\pi}\exp(-\frac{2\pi}{w}d) \tag{20}$$

where d is distance from the wall. For $d=w$ $\underline{distortion=1.2\cdot 10^{-3}\cdot w}$, and for a typical strip width of 2 cm we have distortion of only 24μm!

In order to test this results we can use the exact form of the field for the zero gap and infinite radius derived from conformal mapping

$$u+iv=\ln\left(2\sin\frac{x+iy}{c}\right) \tag{21}$$

This is a very well known mapping used for the calculation of the field and equipotential lines for the wire grid of the MWPC's. In our case we have the complementary problem so that u and v have interchanged their roles and for the field lines we get the relation ($w=1$):

$$y=\frac{1}{\pi}\operatorname{acosh}\sqrt{\frac{e^{2u}}{4}+\cos^2\pi x} \tag{22}$$

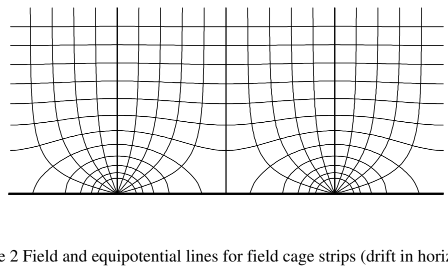

Figure 2 Field and equipotential lines for field cage strips (drift in horizontal direction)

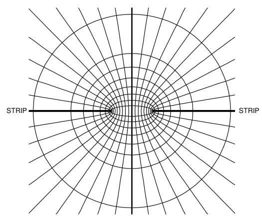

Figure 3 Field lines around the gap

Fig. 2 shows the part the field cage field lines for an infinite small gap (width $2w$ and height $1w$). For a gap of the finite size circular field lines around the gap are deformed into ellipses as shown in Fig. 3. If we assume that drifting electrons follow the field lines, than from relation (22) it follows that maximal distortion at the distance of strip width equals to

$$distortion=1.193318\cdot 10^{-3}w$$

in full agreement with previous results. Fig. 4. shows maximal distortion as a function of the distance from the cage wall. As one can see, contributions of higher terms in the relation (19) are negligible for distances greater than $w/3$. In Fig. 2. we can see that for distances less than $w/3$ the field lines start to end up in the strips which means that the electrons produced in that region are collected on the strips. Relation (22) gives $0.28\cdot w$ for that 'turning point'.

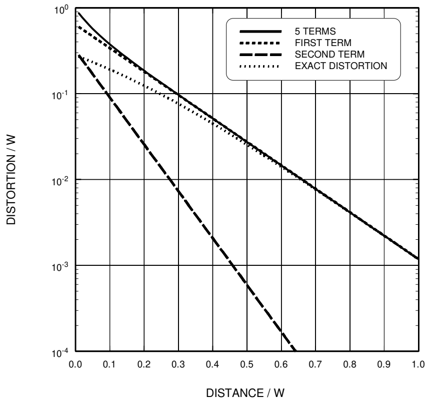

Fig 4 Maximal distortion as a function of the distance from the cage wall.

Because the drifting electrons follow the field lines, one can expect distorsions in z coordinate close to the field cage walls. If we assume that the drift velocity does not change (field intensity constant), then the distortion is equal to the difference between path length for electrons wiggling along the field lines and the z coordinate of the production point. Electrons drifting over the full drift length will have maximal distortion. Fig. 5 shows results of the caculations using first term in equation (19) and exact field lines (22). The path length difference is less than 10μm at the distance from the cage wall equal to the strip width.

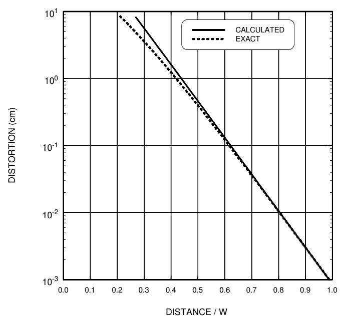

Figure 5. Drift length distortion (in cm) as a function of the distance from the field cage wall for total drift length of 250cm and strip width of 1.2cm

5. Radial Distortion from Resistor Errors in the Resistor Chain

For this calculation the resistors have the uniform random values between 1.99 and 2.01 megaohms (0.5% tolerance). The dimensions of the filed cage are taken to be close to the values for the Alice TPC:

$L=250\ cm$	length of the field cage cylinder (Max drift length)
$\rho_2=250\ cm$	outer cylinder radius
$\rho_1=80\ cm$	inner cylinder radius
$N_s=208$	number of strips (strip width $w=1.2\ cm$)

Results from the previous chapter show that the local distortions of the drift field decrease exponentially as a function of the distance from the field cage wall. Therefore, we can expect that the tolerance in resistor values has much less influence in the magnitude of the distortion than sorting of the resistors.
The error potential was calculated using a linear interpolation of the discrete values given by

$$\phi_k = \frac{\sum_{i=0}^{k} R_i}{\sum_{i=0}^{N_s} R_i} - \frac{k}{N_s}$$

where R_i are randomly produced resistor values and $R_0=0$.
Figures 6. to 9. show error potentials (dots) and calculated error potentials (solid lines) for four different resistors orderings.

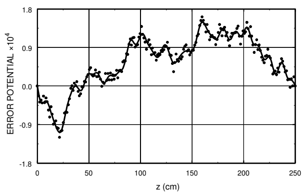

Fig. 6. Error potential for randomly sorted resistors

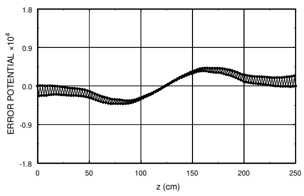

Fig. 7. Error potential for the resistors sorted as complementary pairs

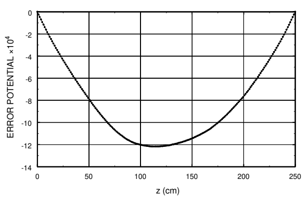

Fig. 8. Error potential for the resistors sorted in ascending order

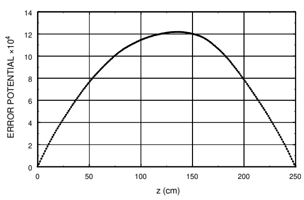

Fig. 9. Error potential for resistors sorted in descending order

For Figures 6, 8, and 9 the calculation was done with 50 terms in the expansion, and for Figure 7 with 450 terms (using fast Fourier transform) in order to reproduce the finest details of the error potential. Factor 10^4 was used for calculation purposes. One can see that for particularly bad sorting of the resistors in ascending or descending order produces a relative error in the potential of the order of 10^{-3}. But, if one sorts resistors as complementary pairs (resistor with minimal resistance, resistor with maximal resistance, etc...) the relative error in the potential can be reduced more than ten times. Calculation of the radial distortions caused by different sorting of the resistors are done for three radial positions: 245cm (5cm away from the outer cage wall), 225cm and 205cm. Results are shown in Figures 10 to 13.

As expected, the ordering of the resistors has a significant effect on the distortion. Sorting resistors as complementary pairs allows to use 0.5% tolerance resistors resulting in radial distortions of only less than 150µm. Next iteration (ordering complementary pairs) would improve that value, but not significantly.

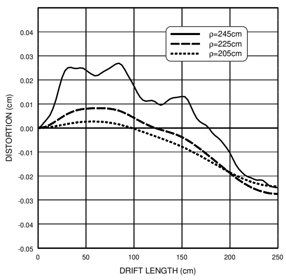

Fig. 10. Radial distortion as a function of drift distance for randomly sorted resistors

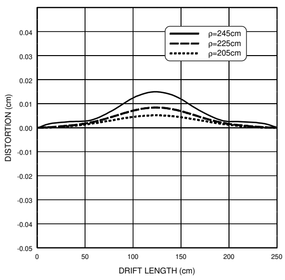

Fig. 11. Radial distortion as a function of drift distance for resistors sorted as complementary pairs

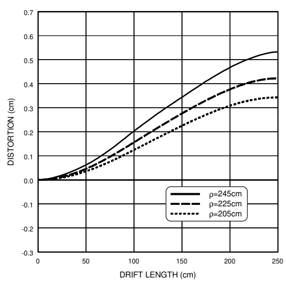

Fig. 12. Radial distortion as a function of drift distance for resistors sorted in ascending order

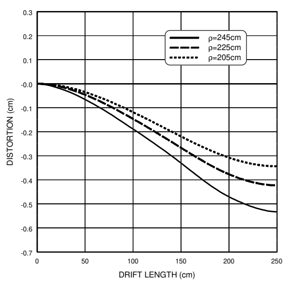

Fig. 13. Radial distortion as a function of drift distance for resistors sorted in descending order

6. Radial Distortion Caused by Shorted Strips

For this serious error in the field cage we can expect large distortions because it would mean a global change of the drift potential. Here we consider a condition where two stripes of the outer field cage wall are connected together. Ignoring the error potential due to the finite size of the strips, we have for the error potential along the outer field cage wall:

$$\phi(z) = \frac{1}{L}\left(\frac{1}{N_s - 1}\right) z \qquad \text{for } z < z_s$$

$$\phi(z) = \frac{1}{L}\left(\frac{1}{N_s - 1}\right) z - \frac{1}{N_s - 1} \qquad \text{for } z > z_s$$

where z_s represents the location of the shorted strips. Substituting this expressions in the equations (14) gives

$$S2_n = -2\frac{\cos\left(n\pi \frac{z_s}{L}\right)}{n\pi\left(N_s - 1\right)} \qquad S1_n = 0$$

Resulting distortion for the full drift volume is shown in Fig. 14.

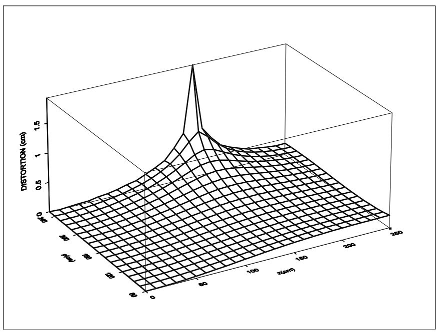

Fig. 14. Distortion resulting from the shorted strips on the outside field cage wall at z=150cm
The sign of the distortion is reversed.

As one can see, this is a rather catastrophic scenario and one has to repair the connected strips, or to deal with the huge distortion corrections (up to more than 1cm!).

7. Distortion Caused by the Endcap Mismatch

The incorrect matching of the end cap potential with the field cage cylinder can be caused by mechanical mismatch or by wrong potential. The following calculation was done for the case when $\delta V=10^{-4}$ corresponding to 10V for 100kV drift voltage, or mechanical displacement of 250μm. Then for the error potential on the inner and outer cylinder we have expression:

$$\phi(z) = \delta V \frac{L-z}{L}$$

and from (14) follows

$$S1_n = S2_n = \frac{2\,\delta V}{n\,\pi}$$

Resulting distortions for different radial distances inside the field cage are given in Fig. 15.

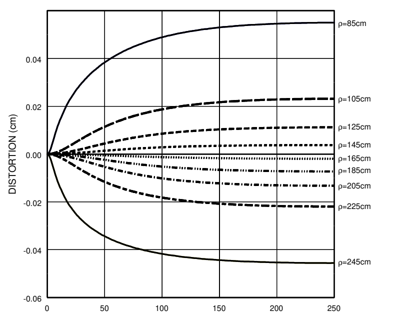

Fig. 15. Distortions for different radial distances resulting from a voltage mismatch of 10^{-4} at the ground plane.

The effective position of the ground plane depends of the gating grid offset voltage and it can be tuned to get minimal distortion, but if the end plane is not parallel to the equipotenital strips within 10^{-4} (corresponding to 250μm over L=250cm) we can expect to have a similar magnitude of the distortion.

8. Distortion Caused by the Endcap Deformation

The exact solution of Laplace equation one can get only for few simple boundary conditions. One of them is for two coaxial cylinders with linear increase of the potential along the z direction and one end side plane (perpendicular to the axis) on a constant potential (as it is the case of the TPC field cage). The second end can be with different length for inner and outer cylinder. From the equation (6) one gets

$$\Phi(\rho, z) = D_1 z \ln(\rho) + D_2 z$$

where

$$D_1 = \frac{\dfrac{V_2}{z_2} - \dfrac{V_1}{z_1}}{\ln\dfrac{\rho_2}{\rho_1}} \qquad\qquad D_2 = \frac{\dfrac{V_2}{z_2}\ln(\rho_2) - \dfrac{V_1}{z_1}\ln(\rho_1)}{\ln\dfrac{\rho_2}{\rho_1}}$$

V_1 and V_2 are voltages applied on the second ends of the cylinders and z_1,z_2 are lengths of the inner and outer cylinders respectively. For $V_1=V_2$ and $z_1=z_2$ we have $D_1=0$ and potential equal, as one would expect,

$$\Phi(\rho, z) = \frac{V_1}{z_1} z$$

independent of ρ. For $V_1 \neq V_2$ or $z_1 \neq z_2$ (or both) we have radial and axial components of the field equal to:

$$E_\rho(\rho, z) = -\frac{\partial\Phi}{\partial\rho} = -D_1 \frac{z}{\rho} \qquad\qquad E_z(\rho, z) = -\frac{\partial\Phi}{\partial z} = -D_1 \ln(\rho) - D_2$$

Inserting this expressions into RDI (1) it is possible to calculate radial distortions in the TPC field cage due to deformed end cap, but the deformation has to have an exactly defined shape in order to satisfy the boundary conditions.
As an example, let $V_1=V_2$ and $z_1=250cm$, $z_2=250.025cm$. Then for the deformation of the end cap (which is at the same time equipotential surface) we get the shape presented in Fig. 16.

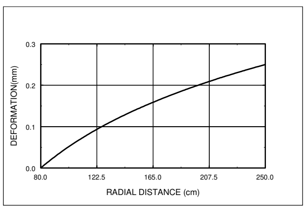

Fig 16. Deformation of the end cap for 250μm difference in the size of the inner (ρ=80cm) and outer (ρ=250cm) cylinder

Resulting radial distortion for several radial distances is given in Fig 17.

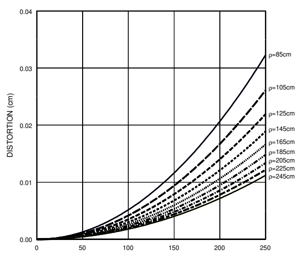

Fig 17 The radial distortion is shown for deformed end cap (HV plane) as a function of the drift distance

The amplitude of the distortion is as expected from the results of the previous chapter.

9. Conclusion

Simple geometry of the cylindrical TPC field cage allows to calculate electron drift distortions due to imperfections using exact solutions of the Laplace equation, or using Fourier expansions for approximate solutions. Where it was possible to compare, both methods give very similar results. The results of the calculation show that maximal distortions due to mechanical imperfections are rughly factor two bigger than the errors in the construction of the field cage. The local deformations in the drift field decrease exponentially with the distance (see Eq. (20)) and are therefore negligible. Special care has to be taken for the alignment of the end caps (ground plane and HV plane) because they introduce global deformation of the drift field. The tolerance on resistor precision can introduce signifficant distortions and special care has to be taken to order resistors in the right way. If resistors are ordered as complementary pairs in respect to their values, the distortion can be reduced by factor 10 to 20 comparing to the worst cases of ordering in descending or ascending order. With proper ordering of the resistors, the distortion can be reduced to less than 200μm for resistors with 0.5% tolerance.

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