Hybrid Decays

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Abstract

The heavy quark expansion of Quantum Chromodynamics and the strong coupling flux tube picture of nonperturbative glue are employed to develop the phenomenology of hybrid meson decays. The decay mechanism explicitly couples gluonic degrees of freedom to the pair produced quarks and hence does *not* obey the well known, but model-dependent, selection rule which states that hybrids do not decay to pairs of $L = 0$ mesons. However, the nonperturbative nature of gluonic excitations in the flux tube picture leads to a new selection rule: light hybrids do not decay to pairs of identical mesons. New features of the model are highlighted and partial widths are presented for several low lying hybrid states.

I. INTRODUCTION

Although explicit nonperturbative gluonic degrees of freedom have been anticipated for many years, no clear experimental evidence for their existence has emerged. The search for nonperturbative glue, in particular as manifested in hybrid mesons, would be greatly facilitated by a rudimentary knowledge of the hybrid spectrum and decay characteristics. Although it appears that lattice estimates of light quenched hybrid masses are forthcoming[1], hadronic decays remain difficult to calculate on the lattice. Thus one is forced to rely on model estimates of the couplings of hybrids to ordinary mesons. Historically, there have been two approaches to such estimates. The first assumes that hybrids are predominantly quark-antiquark states with an additional constituent gluon[2] and that decays proceed via constituent gluon dissociation[3]. The second assumes that hybrids are predominantly quark-antiquark states moving on an adiabatic surface generated by an excited "flux tube" configuration of glue[4]. Decays then proceed by a phenomenological pair production mechanism (the "$^3P_0$ model") coupled with a flux tube overlap[5].

The authors of Refs. [3] noted the existence of a selection rule: TE hybrids do not decay to identical S-wave mesons. A similar observation was made by Isgur, Kokoski, and Paton[5] (hereafter referred to as IKP), *no* hybrids decay to identical S-wave mesons[1]. Although this assertion has achieved the status of dogma in hadron phenomenology, we stress that it is model-dependent. In particular, in the work of IKP the selection rule follows from the assumption that the quark pair production operator is completely decorrelated with the gluonic modes in the hybrid (as it is in the successful $^3P_0$ model[6]). This implies that the hybrid flux tube degrees of freedom must annihilate the final state mesonic flux tubes. The result is a spatial flux tube overlap function which defines the region in which the pair creation may occur, has a node along the hybrid quark-antiquark axis, and leads to the above mentioned selection rule. However, it is entirely feasible for the hybrid flux tube to

[1]Note that the analogue of TE hybrids does not exist in the flux tube model.

be correlated with the pair produced quarks. In this case the argument of IKP breaks down and the selection rule need not exist. This is precisely what happens in the model we develop here.

Our approach is based on the successful description of the Dirac structure of confinement employed in Ref.[7]. The idea is to use the heavy quark expansion of the Coulomb gauge QCD Hamiltonian to identify relevant operators. The gluonic portion of these are then evaluated using a slightly extended version of the flux tube model of Isgur and Paton[4]. In the heavy quark limit the quark and antiquark decouple at leading order. We label these χ and h where $h^\dagger$ creates a quark and χ creates an antiquark. In this approach the interaction contains the kinetic energy term: $H_{int} = -\int h^\dagger \boldsymbol{\sigma}\cdot\mathbf{D}\chi + \mathrm{H.c.}$, where $\mathbf{D} = i\nabla + g\mathbf{A}$. It should be noted that this interaction only contains terms which produce or annihilate a $Q\bar{Q}$ pair (i.e., gluon production or annihilation by a through-going quark line are not present). This implies that contributions to the decay which are higher order in the S-matrix expansion are also higher order in $1/M_Q$. Our model is therefore rather simple: calculate the decay of a (flux tube model) hybrid into two ordinary mesons to first order in H_{int}. The essential new feature is that the gluon field operator should be expressed in terms of the nonperturbative phonon modes of the flux tube model – as opposed to the traditional expansion in plane wave gluons.

II. FLUX TUBE MODEL OF HYBRID DECAYS

As discussed above, we shall employ the flux tube model of Isgur and Paton [4] to construct the hybrid states and the effective decay operator. The model is extracted from the strong coupling limit of the QCD lattice Hamiltonian. The Hamiltonian is first split into blocks of distinct "topologies" (in reference to possible gauge invariant flux tube configurations) and then adiabatic and small oscillation approximations of the flux tube dynamics are made to arrive at an N-body discrete string-like model Hamiltonian for gluonic degrees of freedom. This is meant to be operative at intermediate scales $a \sim b^{-1/2}$ where the strong coupling is of order unity. The lattice spacing is denoted by a and there are N "beads" (or links) evenly spaced between the $Q\bar{Q}$ pair. Diagonalizing the Flux Tube Hamiltonian yields phonons, $\alpha^a_{m,\lambda}$, which are labelled by their color (a), mode number (m), and polarization (λ) which is transverse to the $Q\bar{Q}$ axis. A hybrid may be built of $n_{m\lambda}$ phonons in the m'th mode with polarization $\lambda = \pm$. In particular, hybrid states with a single phonon excitation are constructed as

$$|H\rangle \sim \int d\mathbf{r}\, \varphi_H(r)\, \chi^{PC}_{\Lambda,\Lambda'}\, D^{L_H *}_{M_L,\Lambda}(\phi,\theta,-\phi)\, T^a_{ij}\, b^\dagger_i(\mathbf{r}/2) d^\dagger_j(-\mathbf{r}/2) \alpha^{a\dagger}_{m,\Lambda'}|0\rangle. \tag{1}$$

Spin and flavor indices have been suppressed and color indices are explicit. The factor $\chi^{PC}_{\Lambda,\Lambda'}$ in the hybrid wavefunction projects onto states of good parity and charge conjugation. The quantum numbers of these states[2] are given by $P = \eta_{PC}(-)^{L_H+1}$ and $C = \eta_{PC}(-)^{L_H+S_H+N}$ where $\eta_{PC} = \chi^{PC}_{-1,-1} = \pm 1$ and $N = \sum_m m(n_{m+} + n_{m-})$. We shall consider low-lying hybrids only so that $m = 1$ in what follows.

The definition of the hybrid state makes it clear that decays proceed via the vector potential portion of the covariant derivative (recall that we work at leading order in $1/M_Q$). Thus we proceed by writing the vector potential in terms of phonon operators such that the expressions for the electric and magnetic fields employed in Ref. [7] are recovered in the continuum limit. This yields the following effective decay operator

$$H_{int} = \frac{iga^2}{\sqrt{\pi}} \sum_{m,\lambda} \int_0^1 d\xi \cos(\pi\xi) T^a_{ij}\, h^\dagger_i(\xi \mathbf{r}_{Q\bar{Q}}) \boldsymbol{\sigma} \cdot \hat{\mathbf{e}}_\lambda(\hat{\mathbf{r}}_{Q\bar{Q}}) \left(\alpha^a_{m\lambda} - \alpha^{a\dagger}_{m\lambda}\right) \chi_j(\xi \mathbf{r}_{Q\bar{Q}}), \tag{2}$$

where the $\hat{\mathbf{e}}(\hat{\mathbf{r}})$ are polarization vectors orthogonal to $\hat{\mathbf{r}}$. The integral is defined along the $Q\bar{Q}$ axis only. The integration over the transverse directions yields the factor a^2 which may be interpreted as the transverse size of the flux tube. Note that the phonon operators represent gluonic excitations which are perpendicular to the $Q\bar{Q}$ axis. Although this appears problematical in traditional perturbation theory, it is required here because, in the adiabatic

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[2]These expressions differ from Isgur and Paton[4] because we have adopted the standard definitions for the polarization vectors and the Wigner rotation matrix, following the Jacob-Wick conventions.

limit, the gluonic field configuration must be defined in terms of the quark configuration and therefore the field expansion of the vector potential depends on the quark state under consideration.

The decay amplitude for a hybrid H into mesons A and B is then given by:

$$\langle H|H_{int}|AB\rangle = i\frac{ga^2}{\sqrt{\pi}}\frac{2}{3}\int_0^1 d\xi \int d\mathbf{r}\cos(\pi\xi)\sqrt{\frac{2L_H+1}{4\pi}}\mathrm{e}^{\frac{i\mathbf{p}\cdot\mathbf{r}}{2}}\varphi_H(r)\varphi_A^*(\xi\mathbf{r})\varphi_B^*((1-\xi)\mathbf{r})\cdot \left[\mathcal{D}^{L_H*}_{M_L\Lambda}(\phi,\theta,-\phi)\chi^{PC}_{\Lambda,\lambda}\hat{\mathbf{e}}_\lambda(\hat{\mathbf{r}})\cdot\langle\boldsymbol{\sigma}\rangle\right] \tag{3}$$

where $\langle\boldsymbol{\sigma}\rangle$ is the matrix element of the Pauli matrices between quark spin wavefunctions. This amplitude should be multiplied by the appropriate flavor overlap and symmetry factor.

We note the following general properties of the decay amplitude. The operator is nonzero only along the hybrid $Q\bar{Q}$ axis – as follows from the structure of the interaction Hamiltonian. Thus $q\bar{q}$ creation occurs on a line joining the original $Q\bar{Q}$ quarks, smeared over the transverse size of the flux tube. This is in contrast to the model of IKP which has a node along the $Q\bar{Q}$ axis. Furthermore the spin operator contracts with the flux tube phonon polarization vector, which is absent in the IKP model. Finally, the decay amplitude vanishes when the final mesons are identical for any single-phonon hybrid in an odd mode due to the nodal structure in the vector potential. Thus one obtains the selection rule: low-lying hybrids do not decay to identical mesons. This subsumes the selection rule of IKP so that none of their qualitative conclusions are changed. However we also predict, for example, that hybrids do not decay to pairs of identical P-wave mesons.

III. APPLICATIONS

All strong hadronic decay calculations depend heavily on the hadronic wavefunctions employed. This is especially true here when final states with similar spatial wavefunctions are considered. We have taken the simple approach of adopting simple harmonic oscillator wavefunctions with width parameters chosen to minimize the eigenenergies of a constituent quark model Hamiltonian[8]. Several of the predicted widths were compared to calculations with exact wavefunctions – the results differed by less than 10%, leading us to expect that the widths presented below are reasonably reliable. Nevertheless dramatically different wavefunction width parameters have been used in the literature (cf., the third reference of Ref. [6]) so that the reader is cautioned that these results are qualitative (pending more detailed analysis[9]).

In the following, the normalization is fixed to yield a total width for the $\hat{\pi}(1.8)$ of 220 MeV, close to the experimental value of 212 ± 37 MeV for the $\pi(1.8)$. This gives $ga^2 = 2.56$ GeV^{-2}. Notice that this is consistent with the strong coupling relationship which relates the string tension to these parameters, $b = g^2 C_F / 2a^2$, if one assumes $g \sim 1$. Table I presents the decay widths for isoscalar, isovector, and $s\bar{s}$ hybrids with $J^{PC} = 1^{-+}$, 0^{-+}, and 1^{--}.

TABLES

TABLE I. Dominant Hybrid Decay Modes.

state	J^{PC}	mode	partial waves	partial widths (MeV)
I=1 $\hat{x}(1.9)$	1^{-+}	πb_1	S,D	87, 4
		$K\bar{K}_1(1.4)$ [a]	S,D	24, 0
		πf_1	S,D	21, 0
		$\pi\rho$	P	11
		ηa_1	S,D	9, 0
		$K\bar{K}_1(1.27)$ [a]	S,D	8, 0
		$K\bar{K}^*$	P	2
I=0 $\hat{y}(1.9)$	1^{-+}	πa_1	S,D	64, 1
		$K\bar{K}_1(1.4)$	S,D	24, 0
		ηf_1	S,D	8, 0
		$K\bar{K}_1(1.27)$	S,D	7, 0
		$K\bar{K}^*$	P	2
$s\bar{s}$ $\hat{z}(2.1)$	1^{-+}	$K\bar{K}_1(1.4)$	S,D	99, 0
		$K\bar{K}_1(1.27)$	S,D	13, 0
		$K\bar{K}^*$	P	7
I=1 $\hat{\rho}(1.9)$	1^{--}	πa_1	S,D	87, 1
		$K\bar{K}_1(1.27)$	S,D	13, 0
		$\pi\omega$	P	10
		$K\bar{K}_1(1.4)$	S,D	7, 0
		$\eta\rho$	P	5
		$\eta'\rho$	P	1
		πa_2	D	1

+I=0 $\hat{\omega}(1.9)$	1^{--}	$\pi\rho$	P	33
		$K\bar{K}_1(1.27)$	S,D	13, 0
		$K\bar{K}_1(1.4)$	S,D	7, 0
		$\eta\omega$	P	5
		$\eta'\omega$	P	1
$s\bar{s}$ $\hat{\phi}(2.1)$	1^{--}	$K\bar{K}_1(1.4)$	S,D	53, 0
		$K\bar{K}_1(1.27)$	S,D	23, 0
		$\eta\phi$	P	10
		$\eta'\phi$	P	3
		$K_2^*\bar{K}$	D	1
I=1 $\hat{\pi}(1.8)$	0^{-+}	$\bar{K}(K\pi)_{K_0^*}$ [b]	S	85
		$\pi f_0(1.3)$	S	73
		$\pi\rho$	P	38
		$\eta a_0(1.3)$	S	14
		$K\bar{K}^*$	P	6
		$\pi\rho(1.465)$	P	4
		πf_2	D	1
I=0 $\hat{\eta}(1.8)$ [c]	0^{-+}	$\pi a_0(1.3)$	S	217
		$\bar{K}(K\pi)_{K_0^*}$	S	85
		$\eta f_0(1.3)$	S	14
		$K\bar{K}^*$	P	6
		πa_2	D	2
$s\bar{s}$ $\hat{\eta}'(2.0)$ [c]	0^{-+}	$K\bar{K}_0^*$	S	293

$K\bar{K}^*$	P	26
$K\bar{K}_2^*$	D	12

[a]A mixing angle of 34 degrees has been used for the $K_1(1.27)$ and $K_1(1.4)$.

[b]The width of the K_0^* has been incorporated by convoluting with the appropriate Breit-Wigner formula.

[c]The flavor structure of the $\hat{\eta}$ and $\hat{\eta}'$ are taken to be $(u\bar{u}+d\bar{d})/\sqrt{2}$ and $s\bar{s}$ respectively.

IV. CONCLUSIONS

We have used an adiabatic approximation to hybrid structure to study the decay of gluonic hybrid mesons. Hybrids may be considered as quarks moving on an adiabatic surface generated by a flux tube of nonperturbative glue in an excited state. In the heavy quark limit of QCD the decay operator is given by the transverse gluon component of the covariant derivative. In keeping with the spirit of the flux tube model of Isgur and Paton, we chose to evaluate the gluonic portion of the decay amplitude in terms of phonon degrees of freedom. The resulting expression differs significantly from that of Ref. [5] – but surprisingly gives similar predictions for the total widths of hybrids. The main difference seems to be in substantially smaller D-wave amplitudes predicted here (this is reminiscent of the difference between the $^3P_0$ model and the $^3S_1$ model; see the last paper of Ref. [6]). Qualitatively, the calculation presented here obeys the well-known spin selection rule, spin zero states do not decay to pairs of spin zero states, and a new rule: low lying hybrids do not decay to pairs of identical mesons. Decays to mesons with similar spatial wavefunctions are also predicted to be suppressed.

Several candidate hybrid states exist. One of these is the $\pi(1800)$ seen in $\pi f_0(980)$, $\pi f_0(1300)$, and $K(K\pi)_S$[10] decay channels. Significantly, the channels $\pi\rho$ and KK^* are suppressed. Although a 3S quarkonium state is expected near 1.8 GeV, the decay characteristics support the identification of this state as a hybrid (see the discussion in Ref. [11] for a similar analysis). In particular the π_{3S} is expected to decay strongly to $\omega\rho$ while this mode is practically zero in our approach. The strength seen in $\pi f_0(1300)$ is in accord with Table I and is again in conflict with a $q\bar{q}$ identification of the $\pi(1800)$. The interpretation of the $\pi f_0(980)$ mode is complicated by the large coupling of the $f_0(980)$ to the $K\bar{K}$ channel. Indeed, it is likely that the $f_0(980)$ is a $K\bar{K}$ bound state stablized by the $f_0(1300)$[12]. In view of this, the large $\pi f_0(980)$ mode may be due to strong final state interactions in $K\bar{K}_0^* \to K(\bar{K}\pi)$. A search for isopartner states $\hat{\eta}$ and $\hat{\eta}'$ which are broad (roughly 330 MeV) and which do not couple to $\rho\rho$ or $\omega\omega$ will be very instructive.

Strong evidence for hybrids would be obtained if the 1^{-+} seen at 1.9 GeV at BNL[13] is confirmed since these quantum numbers are exotic. This state has been seen in πb_1 and πf_1 in accordance with our expectations; however, we also expect a reasonably large $K\bar{K}_1(1.4)$ mode and suggest that this should be investigated (although this might be complicated by strong $K\bar{K}$ final state interactions). Finally, we consider the prospect for vector hybrid production at TJNAF. If one assumes $\hat{\rho}$ and $\hat{\omega}$ production by vector meson dominance followed by t-channel π exchange then isospin symmetry implies that the ratio of cross sections is $\sigma(\hat{\omega})/\sigma(\hat{\rho}) = 27$. Thus it would be expedient to search for the isoscalar vector hybrid. This is an interesting state because all of the normally large S+P channels are excluded by quantum numbers and it is therefore very narrow, with a predicted width of roughly 60 MeV. The predominant decay mode is $\pi\rho$; we therefore suggest that the search for the $\hat{\omega}$ be conducted in the $\pi^0\pi^+\pi^-$ final state.

ACKNOWLEDGMENTS

We are grateful to Ted Barnes, Suh Urk Chung, and Alex Dzierba for discussions on the phenomenology of hybrid states. ES acknowledges the financial support of the DOE under grant DE-FG02-96ER40944.

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